The Role of Multinational Production
in Cross-Country Risk Sharing*

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Abstract

In this paper, we introduce the role of Multinational Production (MP) in cross-country risk sharing. We present a two-country, two-sector model with complete financial markets, and country-specific productivity shocks to the non-tradable sector. Firms can do MP by opening affiliates abroad which bear the productivity shock to the host country. In a world with asymmetric countries, MP improves the scope for international diversification beyond the existence of a full set of contingent claims. This result stems from treating MP simultaneously as a portfolio and production flow. By changing total factor productivity in the host country, MP affects the global impact of country-specific productivity shocks. The model has predictions on the composition of international portfolios across countries. We calibrate the model to US business cycle and external account moments to quantify this new role of MP. We find that not accounting for the reallocation of production entailed by direct investment flows may underestimate by around 50% the effect of changes in world volatility on the US net direct investment position.

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1 Introduction

When analyzing the composition of countries’ international portfolio, the literature on international risk sharing has focused on the distinction between risky and risk-free assets, without differentiating Foreign Direct Investment (FDI) from other risky positions. The vast majority of such macroeconomic models takes the market structure of goods and factors as given. Under such restriction, buying shares of foreign firms or doing FDI are indeed equivalent. However, the crucial difference between FDI and other international flows is that the former involves reallocation of production. Indeed, one of the most notable features of economic globalization has been the increasing importance of Multinational Production (MP) in international goods markets: by 2004, total sales of foreign affiliates of multinational firms represented 51% of world GDP, almost double the share of world exports.\(^1\)

In this paper we introduce the role of MP both as a production and portfolio flow in cross-country risk sharing. We find that the change in the host country’s Total Factor Productivity (TFP) entailed by FDI flows has implications for the pattern of world aggregate risk. We also find that international risk patterns affect the location of multinational firms and the international portfolio composition of countries. In particular, “large” economies have further incentives to do MP –where “large” refers to the magnitude of the impact of country-specific shocks on world financial prices–. As a counterpart, these countries tend to have a larger FDI position.

Treating MP simultaneously as a portfolio and production flow results in important and novel insights. First, it fundamentally alters previous results on the relevance of MP in international risk-sharing: if the impact of MP on the host countries’ TFP is ignored, MP flows only affect international risk-sharing under imperfect financial markets. By contrast, in the framework proposed here, MP flows have a role in international risk-sharing even if a complete set of state contingent bonds exists, as it alters the international production structure.

\(^1\)World Investment Report 2006, UNCTAD.
Second, the interaction between these two roles of MP results in novel cross-country implications for the location of firms. We find that for large economies these two roles of MP as a portfolio and production flow are complements: risk considerations increase incentives of firms from large countries to do MP while the opposite happens for smaller economies.

We present a two-country stochastic model with a full set of contingent financial assets. There are two sectors: tradable and non-tradable. We introduce country-specific shocks that affect the relative price of tradable to non-tradable goods. In the spirit of Melitz (2003), firms in the non-tradable sector are heterogenous, compete monopolistically, and can do MP by opening affiliates abroad after paying an entry cost. Affiliates bear the productivity shock to the host country. Hence, with elastic demand functions, MP profits co-move with host country risk.

Unlike the rest of the literature on international risk-sharing, we do not differentiate assets according to their risk. Rather, our analysis distinguishes the following two international assets: FDI, which involves changes in relative TFP across countries; and a financial portfolio of other risky and risk-free assets, that we reinterpret as fully contingent claims. The representative consumer holds Arrow-Debreu securities and shares of national firms that include multinationals.\footnote{The results are not affected if national firms are initially owned by national consumers and later sold in the international market.}

We recognize that countries differ in the impact of their shocks on world output. For instance, if countries only differ in the size of their productive sector, a relative bad shock to the larger economy has bigger impact on world output. Hence, world risk co-moves with the one of the larger country. In other words, if countries are asymmetric, productivity shocks cannot be fully diversified even in a world with frictionless financial markets. In this context, reallocation of production from larger to smaller economies improves the scope for risk diversification as it brings economies closer together in terms of their contribution to world output.

The role of MP as a risk-sharing device comes from the fact that profits of affiliate plants co-
move with host country risk. It follows that MP profits of multinational firms from the smaller country are higher when world output is abundant, while profits of multinational firms from the larger economy are higher when world output is more scarce. In that sense, MP from the larger economy is more valuable in terms of risk sharing. As a result, firms from the larger economy have further incentives to do MP.

To quantify the impact of risk on reallocation of production and the portfolio decision of a country, we calibrate the model to business cycle and external account moments for the US economy and an aggregate of Rest of the World. We decompose the period 1960-2005 into two sub-periods, 1960-1984 and 1985-2005, during which the US experienced a reduction in GDP volatility from 2.9% to 1.7%, while the Rest of the World went from 2.9% to 1.1%. We compare two steady states: one consistent with the volatility in 1960-1984, and the second with the volatility in 1985-2005. Crucial to our results is the finding that world financial prices – understood as the price of the Arrow-Debreu securities- follow the US risk in the second steady state but not in the first one. This feature implies important differences in terms of international portfolio composition between these two steady states. US net direct investment position (measured as discounted flows of profits) increases from 34% to 72% of GDP, while the net position on other assets goes from -0.03% to -10% of GDP. The endogenous reallocation of production triggered by this change in volatilities accounts for half of the total change in the US net direct investment position.

Our model builds on Obstfeld and Cole (1992), Backus and Smith (1993), and the literature on international risk-sharing. As in that literature, contingent claims have a role when there are non-tradable goods. The economy represented here has complete financial markets, which guarantees perfect international risk sharing up to the existence of non-tradable goods and a given production structure –i.e. the process of cross-country TFP shocks–. Differently from that literature, we find that MP has a role in international risk sharing beyond the existence of a
complete set of state contingent claims, as it alters the impact of different country productivity shocks on overall world output.

To our knowledge, no study has analyzed the role of MP in international risk-sharing treating it simultaneously as a portfolio and production flow.\(^3\) On the one hand, the international trade literature has focused on the role of MP as a way of serving foreign consumers by replicating production facilities abroad (horizontal FDI), or splitting the production chain to take advantage of cheap input costs (vertical FDI).\(^4\) This literature emphasizes the role of MP in the exchange of goods but does not address its implications in terms of international risk-sharing. On the other hand, the international business cycle literature has mainly treated MP as a portfolio flow abstracting from the production reallocation that this flow entails.\(^5\) This disconnection between international macroeconomics and trade misses interesting and relevant synergies.

The paper has the following structure. Section 2 presents the model: the set-up, the equilibrium conditions, and the main mechanism. Section 3 presents the calibration and counterfactual exercises. Section 4 concludes.

2 Model

The model in this section highlights the role of MP as a risk sharing device across countries. This is a stochastic, two-period, two-country model with complete financial markets. There are two sectors: tradable and non-tradable. The non-tradable sector is subject to a country-specific

\(^3\)Aizenman and Marion (2004), and Goldberg and Kolstad (1995) study the location of MP activities under uncertainty. Both frameworks and motivations are very different from ours. They do not address the change in aggregate risk that results from reallocation of production nor do they have financial assets that allow firms to optimally diversify risk.

\(^4\)See Markusen (1984); Brainard (1997); Markusen and Venables (1998); Carr, Markusen, and Maskus (2001); Helpman, Melitz, and Yeaple (2004); Ramondo (2005), Burstein and Monge (2006), for horizontal FDI. See Helpman (1984), Antras (2003), Antras and Helpman (2004), for vertical FDI.

productivity shock.

Firms in the non-tradable sector are heterogenous in productivity. They can do MP by opening affiliates abroad, after paying an entry cost. Affiliates inherit the productivity parameter from the source firm but bear the shock to the non-tradable sector in the *host* country. Hence, MP profits co-move with host country risk.

Consumers hold shares of national firms that include multinationals, and Arrow-Debreu securities.

2.1 Set-up

There are two countries, Home and Foreign, of size $L$ and $L^*$, respectively. Firms can do MP by opening affiliates abroad. Hence, it is relevant to distinguish between national (ownership criteria) and domestic (location criteria) variables. We indicate with an asterisk * those variables that are owned by Foreign consumers, irrespectively of location.

There are two periods: an initial period, *before* country-shocks are realized, in which trade in Arrow-Debreu securities and Foreign Direct Investment (FDI) take place; and a second period, *after* uncertainty is realized, in which production and consumption take place.

Let the vector $s \in S$ denote the state of the world economy in this second period, which is characterized by the realization of country productivity shocks. Assume that there is a finite discrete number of states: $S = \{s_1; s_2; ...; s_n\}$, each with probability $\Pr(s)$, $\sum_{s \in S} \Pr(s) = 1$, and $0 \leq \Pr(s) \leq 1$.

Each economy produces two types of goods: an aggregate CES non-tradable consumption good

$$C_{NT}(s) = \left[ \int_{\omega \in \Omega} c(\omega)^{\frac{\eta-1}{\eta}} d\omega \right]^{\frac{\eta}{\eta-1}},$$

(1)
with elasticity of substitution $\eta > 1$, and price index:

$$P_{NT}(s) = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\eta} d\omega \right]^{\frac{1}{1-\eta}},$$  \hspace{1cm} (2)

and a freely-traded homogenous consumption good $C_T(s)$ that is used as numeraire, $P_T = 1$.

**Preferences.** The representative consumer supplies $L$ units of labor and maximizes the following expected utility from consumption:

$$U = \beta \sum_{s \in S} \Pr(s) u(C(s)),$$

where

$$C(s) = \left[ C_T(s) \frac{1-\rho}{\rho} + C_{NT}(s) \frac{1-\rho}{\rho} \right]^{\frac{\rho}{\rho-1}},$$  \hspace{1cm} (4)

$\rho > 1$. The price index for $C(s)$ is:

$$P(s) = \left[ 1 + P_{NT}(s)^{1-\rho} \right]^{\frac{1}{1-\rho}}.$$  \hspace{1cm} (5)

Total expenditure in each individual good $\omega$ is

$$x(\omega) = \left[ \frac{p(\omega)}{P_{NT}(s)} \right]^{\frac{1-\eta}{\eta}} X_{NT}(s),$$  \hspace{1cm} (6)

where $X_{NT}(s)$ is aggregate expenditure in the CES good:

$$X_{NT}(s) = \left( \frac{P_{NT}(s)}{P(s)} \right)^{1-\rho} X(s)$$

and $X(s)$ is aggregate expenditure. Total expenditure in the homogeneous good is:

$$X_T(s) = \left( \frac{1}{P(s)} \right)^{1-\rho} X(s)$$
Technology. There is a continuum of firms of measure one, each producing a differentiated good $\omega$. Each firm operates an only-labor constant returns to scale technology with productivity $z(\omega)$. The parameter $z(\omega)$ is known, and drawn from a country-specific distribution, $G(z)$ and $G^*(z)$, for Home and Foreign respectively, independently distributed across countries.

Additionally, firms are subject to a country-specific, aggregate productivity shock, $A$ and $A^*$, which is the only source of risk in this economy. Hence, $s = \{A, A^*\}$.

Firms can open affiliate plants abroad with the same productivity parameter $z(\omega)$ as the one they have at home. Hence, production functions for a Home firm producing good $\omega$ at Home and Foreign are, respectively:

$$y_d(\omega, s) = A \cdot z(\omega) \cdot l(\omega, s)$$  \hspace{1cm} (7)

$$y_m(\omega, s) = A^* \cdot z(\omega) \cdot l^*(\omega, s).$$  \hspace{1cm} (8)

where $y_d(\omega, s)$ and $y_m(\omega, s)$ are domestic and foreign output, while $l(\omega, s)$ and $l^*(\omega, s)$ are labor requirements. Since the only parameter that varies across differentiated goods is the firm-specific productivity $z(\omega)$, and goods enter symmetrically in preferences, we can rename each good $\omega$ by its productivity $z$. Total profits of a Home firm with productivity $z$ are given by:

$$\pi(z, s) = \pi_d(z, s) + \tau(z)\pi_m(z, s),$$  \hspace{1cm} (9)

where $\tau(z)$ is one if the firm does MP and zero otherwise, $\pi_d(z, s)$ denotes profits at Home, and $\pi_m(s, z)$ profits in Foreign.

Firms compete monopolistically. Hence, the price charged by a firm with productivity $z$ at Home is:

$$p(z, s) = \frac{\eta}{\eta - 1} \cdot \frac{W}{A} \cdot \frac{1}{z},$$  \hspace{1cm} (10)
and in Foreign:

\[ p(z, s) = \frac{\eta}{\eta - 1} \cdot \frac{W^*}{A^*} \cdot \frac{1}{z} \]  

(11)

The tradable homogeneous consumption good is produced under constant returns to scale with an only-labor technology and productivity \( W: Y_h(s) = W L_h(s) \). Provided that this good is produced everywhere, nominal wages at Home and Foreign are, respectively, \( W \) and \( W^* \).\(^6\)

**Assets Structure.** The representative consumer in each country holds two types of assets: shares of firms, \( \theta(z) \) and fully contingent bonds \( B(s) \). Firms are assumed to be owned by national consumers \( \theta(z) = 1 \) for \( z \in Z \) and \( \theta^*(z) = 1 \) for \( z \in Z^* \).\(^7\) The budget constraint is therefore:

\[
\sum_{s \in S} q(s) P(s) C(s) = B_0 + \sum_{s \in S} q(s) \left\{ LW + \int_{z \in Z} \theta(z) \pi(z, s) dG(z) \right\}
\]  

(12)

where \( q(s) \) is the date-0 price of an Arrow-Debreu security that pays one unit of the numeraire in state \( s \), and \( B_0 \) is initial net wealth. Finally \( \pi(z, s) \) denotes profits of Home firms with technology \( z \). From the consumer’s optimization problem, the Euler equation for securities is:

\[
q(s) = \frac{1}{\lambda} \frac{u'(C(s))}{P(s)} \beta Pr(s)
\]  

(13)

where \( \lambda \) is the multiplier on the Home consumer’s budget constraint.

**Foreign Direct Investment.** Foreign Direct Investment (FDI) takes place in the following way.\(^8\) Firms decide whether to become multinationals or not before the realization of country shocks. If they decide to enter the foreign market, they pay a one time entry cost, \( f \) and \( f^* \), for

\(^6\)It will never be optimal to do MP in this sector.
\(^7\)The results are not affected if national firms are initially owned by national consumers and sold in the international market.
\(^8\)Note the distinction between FDI and MP: the first one refers to the Balance of Payment flow and in our model occurs only once, i.e. the initial setting-up of affiliates abroad; the second one refers to the productive activities of affiliates abroad, e.g. sales, profits, employment, and occurs every period.
Home and Foreign, respectively. The value of doing MP for a firm with productivity $z$ is given by the expected discounted stream of profits $\sum_{s \in S} q(s) \pi_m(z, s)$. Both countries are endowed with an initial stock of an investment tradable good, $Y_0$ and $Y_0^*$. The MP entry cost is paid in units of this good, which international price is denoted by $p_f$. Therefore, the FDI decision is characterized by the following rule:

$$\sum_{s \in S} q(s) \pi_m(z, s) \geq f^* p_f : \tau(z) = 1$$
$$\sum_{s \in S} q(s) \pi_m(z, s) < f^* p_f : \tau(z) = 0.$$  

Finally, the initial net wealth in the budget constraint (12) is given by

$$B_0 = Y_0 - f^* p_f \int_{z \in \mathbb{Z}} \tau(z) dG(z). \quad (14)$$

### 2.2 Equilibrium

We define the equilibrium in two steps. First, we characterize national equilibrium prices and quantities as functions of the state vector $s$, the number of firms doing MP, and aggregate expenditure at Home and Foreign, $X(s)$ and $X^*(s)$. In the second step, we define the international equilibrium.

#### 2.2.1 National Equilibrium

As it is explained in the next subsection, the FDI decision follows a cut-off rule. We denote $\bar{z}$ (and $\bar{z}^*$ for Foreign) the productivity level for which firms with $z$ above $\bar{z}$ become multinationals,
and firms with \( z \) below \( \bar{z} \) stay domestic:

\[
\forall z \geq \bar{z} : \tau(z) = 1 \\
\forall z < \bar{z} : \tau(z) = 0.
\]

The *national equilibrium* prices and quantities are characterized as functions of the state vector \( s \), the cut-off rule for doing MP activities, \( \bar{z} \) and \( \bar{z}^* \), and aggregate expenditure at Home and Foreign, \( X(s) \) and \( X^*(s) \).

Define the following aggregate productivity indexes:

\[
Z_d \equiv \int_{z_{\text{min}}}^{\infty} z^{\eta-1} dG(z) \quad (15) \\
Z_m \equiv \int_{\bar{z}}^{\infty} z^{\eta-1} dG(z), \quad (16)
\]

and analogously for Foreign firms, \( Z_d^* \) and \( Z_m^* \).

From (2), (10), and (11), price indices for the composite good, at Home and Foreign, are given by:

\[
P_{NT}(s) = \frac{\eta}{\eta - 1} \cdot \frac{W}{A} \cdot (Z_d + Z_m^*)^{\frac{1}{1-\eta}} \quad (17) \\
P_{NT}^*(s) = \frac{\eta}{\eta - 1} \cdot \frac{W^*}{A^*} \cdot (Z_d^* + Z_m)^{\frac{1}{1-\eta}}. \quad (18)
\]

The only source of uncertainty in this model comes from the realization of the productivity shock, which only affects the non-tradable sector.

Notice from equations (17) and (18) that the impact of such a shock on the relative price of tradable to non-tradable goods depends on the number of firms operating in that market, which is measured by the aggregate productivity indexes \( Z_d \) and \( Z_m^* \). In other words, the location
of firms alters both average TFP in the host country and the impact of the country specific productivity shocks on the relative price between tradables and non-tradables. This channel introduces the role of MP in international risk sharing.\(^9\),\(^10\)

Profits for an individual Home firm with productivity \(z\) at Home are given by:

\[
\pi_d(z, s) = \frac{1}{\eta} \cdot \frac{z^{\eta-1}}{Z_d + Z_m^*} \cdot X_{NT}(s)
\]  

(19)

and in Foreign:

\[
\pi_m(z, s) = \frac{1}{\eta} \cdot \frac{z^{\eta-1}}{Z_d^* + Z_m} \cdot X_{NT}^*(s).
\]  

(20)

Hence, aggregate profits for domestic and multinational firms from Home are given by:

\[
\Pi_d(s) = \int_{z_{min}}^{\infty} \pi_d(z, s) dG(z) = \frac{1}{\eta} \cdot \frac{Z_d}{Z_d + Z_m^*} \cdot X_{NT}(s)
\]  

(21)

\[
\Pi_m(s) = \int_{z}^{\infty} \pi_m(z, s) dG(z) = \frac{1}{\eta} \cdot \frac{Z_m}{Z_d^* + Z_m} \cdot X_{NT}^*(s).
\]  

(22)

Analogous expressions characterize aggregate profits of Foreign firms.

Profits of multinational firms, \(\Pi_m(s)\), follow the evolution of total expenditure in non-tradable goods in the host market, \(X_{NT}^*\). Since consumption is a CES bundle of homogenous and composite goods with elasticity \(\rho > 1\), total expenditure \(X_{NT}^*(s)\) increases in states where \(P_{NT}^*(s)\) is lower.\(^11\) That is, MP profits of Home multinationals are higher in those states where the host productivity shock, \(A^*\), is large.

\(^9\)If the relative price of tradable to non-tradable goods were constant across states, perfect international diversification would be attained without MP. In such case, the Arrow-Debreu prices in (13) would equalize the ratio of marginal utilities across states. Although MP still would have a wealth effect, its role in risk diversification would be redundant.

\(^10\)Notice that the real exchange rate is \(P(s)/P^*(s)\). From (5), the real exchange rate is lower in states where \(A\) is higher relative to \(A^*\).

\(^11\)In the Cobb-Douglas case, \(\rho = 1\), income and substitution effects cancel out and \(X_{NT}^*\) remains constant across states.
Since differentiated goods are non-tradable, equilibrium for each good \( z \), is given by the feasibility constraint in state \( s \):

\[
y(z, s) = c(z, s).
\]  
(23)

From (6), (7), (8) and (23), aggregate labor demands in the non-tradable sector, for national and foreign firms at Home, are:

\[
L_d(s) = \frac{\eta - 1}{\eta} \cdot \frac{1}{W} \cdot \frac{Z_d}{Z_d + Z_m} \cdot X_{NT}(s)
\]

\[
L^*_m(s) = \frac{\eta - 1}{\eta} \cdot \frac{1}{W} \cdot \frac{Z_m^*}{Z_d + Z_m^*} \cdot X_{NT}(s)
\]

Labor demand in the homogeneous good sector at Home follows from the labor resource constraint:

\[
L = L_h(s) + L_d(s) + L^*_m(s)
\]

Analogous condition characterizes the labor market in Foreign.

### 2.2.2 International Equilibrium

FDI occurs before uncertainty is realized. From (20), MP profits increase in \( z \). Therefore:

\[
\sum_{s \in S} q(s) \frac{\partial}{\partial z} \pi_m(z, s) > 0
\]  
(24)

That is, expected MP profits increase in \( z \). The optimal MP entry decision is therefore given by a cut-off rule, characterized by a productivity level \( \overline{z} \) for which firms are indifferent between
becoming multinationals or not:

$$\sum_{s \in S} q(s)\pi_m(\overline{z}, s) = f^* \cdot p_f$$  (25)

$$\sum_{s \in S} q(s)\pi_m(\overline{z}^*, s) = f \cdot p_f,$$  (26)

where \(p_f\) is the world price of the investment good, and it can be interpreted as the equilibrium price that clears the FDI market. As long as there exists a positive entry cost \(f\), only the most productive firms do MP. It follows from (24) that expected MP profits net of entry cost increase in \(z\).

The national equilibrium prices and quantities can be all characterized as functions of the state vector \(s\), the cut-off rule for doing MP activities, \(\overline{z}\) and \(\overline{z}^*\), and aggregate expenditure at Home and Foreign, \(X(s)\) and \(X^*(s)\). Replacing these functions in (12), the aggregate budget constraint can be re-written as the Balance of Payment condition. We can now close the model and define the international equilibrium as follows:

**Definition.** For a given initial wealth, \(Y_0\) and \(Y_0^\ast\), the international equilibrium is a vector \([X(s), X^*(s), B(s), B^*(s)]\), for each \(s \in S\), a pair \(\{\overline{z}, \overline{z}^*\}\), and prices \([p_f, \{q(s)\}_{s \in S}]\) such that:

1. Arrow-Debreu prices satisfy equation (13) for both countries;

2. The zero profit conditions for MP in equations (25) and (26) are satisfied;

3. The Arrow-Debreu securities are in zero net supply:

$$B(s) + B^*(s) = 0$$
4. The world resource constraint at the initial investment period is satisfied:

\[ Y_0 + Y_0^* = [1 - G(\tau)] f^* + [1 - G^*(\tau^*)] f; \]

5. The intertemporal budget constraint (12) for Home and Foreign is satisfied.

6. The resource constraint for the homogeneous tradable good holds, for each \( s \):

\[ C_T(s) + C_T^*(s) = Y_T(s) + Y_T^*(s) \] (27)

### 2.3 Main Mechanism

In this subsection, we describe the main mechanism of the model. The economy represented here has complete financial markets, which guarantees perfect international risk sharing up to the existence of non-tradable goods and a given production structure –i.e. the number of firms operating in each market-. We find that MP has a role in international risk sharing beyond the existence of a complete set of state contingent bonds, as it alters host country’s TFP and the impact of country specific productivity shocks on global output. To emphasize this new role of MP, we focus on the case where country-specific shocks to the non-tradable sector can take only two values, and are symmetric and perfectly negatively correlated across countries.

The existence of non-tradable goods prevents Purchasing Power Parity (PPP) from holding. Indeed, each state of nature \( s \) is characterized by a different real exchange rate –i.e. \( P(s)/P^*(s) \). From (13), it follows that the ratio of marginal utilities across countries is not constant across states:

\[ \frac{u'(C(s))}{u'(C^*(s))} = \frac{\lambda}{\lambda^*} = \frac{P(s)}{P^*(s)} \]

where \( \lambda \) and \( \lambda^* \) correspond to the Lagrange multipliers on the budget constraint for Home and
When countries are symmetric, full risk sharing is attained -up to the existence of non-tradable goods-. In this case, the productivity shock symmetrically affects the two economies:

$$\frac{u'(C(s_1))}{P(s_1)} = \frac{u'(C^*(s_2))}{P^*(s_2)},$$

where $s_1 = \{A_L, A^*_H\}$ and $s_2 = \{A_H, A^*_L\}$. It follows from (13) that Arrow-Debreu prices are constant across states, which implies that the ratio of marginal utility to the price index is also constant across states:

$$\frac{u'(C(s_1))}{P(s_1)} = \frac{u'(C(s_2))}{P(s_2)}.$$

However, when countries are asymmetric -e.g. $E(A) > E(A^*)$ or $E(z) > E(z^*)$-, a full set of state contingent securities is not sufficient to attain full risk sharing; even if shocks are perfectly negatively correlated across countries, there is some undiversifiable risk. In this case, Arrow-Debreu prices are higher in states where the negative shock hits the more productive economy. Therefore:

$$\frac{u'(C(s_1))}{P(s_1)} > \frac{u'(C(s_2))}{P(s_2)}.$$

The following proposition formalizes this statement:

**Proposition 1.** For $\frac{\sigma}{(\sigma - 1)} < (\eta - 1) / (\rho - 1)$: if $E(A) > E(A^*)$ and countries are otherwise identical, then the steady state Arrow-Debreu prices in a world economy without MP are higher in $s_1$ than in $s_2$, where $s_1 = \{A_L, A^*_H\}$ and $s_2 = \{A_H, A^*_L\}$.

*Proof. See Appendix*

In a world with asymmetric countries, MP has a role in international risk sharing beyond
the existence of a complete set of state contingent securities, as it alters average TFP in the host country. The effect of MP on TFP can be observed in equations (17) and (18): aggregate productivity in the non-tradable sector is given by $A (Z_d + Z_m^*)^{\frac{1}{\eta - 1}}$ for Home, and $A^* (Z_d^* + Z_m)^{\frac{1}{\eta - 1}}$ for Foreign. This expression includes the country productivity ($A$) and an aggregate index of firm-specific productivity, that includes domestic firms $Z_d$ and foreign firms $Z_m^*$. Hence, when $E(A) > E(A^*)$, countries become more similar –and the gap in marginal utilities across states narrows– if $Z_m > Z_m^*$. In other words, if production reallocates towards the less productive country, the productivity gap narrows, increasing the scope for international risk diversification.

The intuition behind this result is the following. Arrow-Debreu prices are higher when Home (i.e. the more productive country) is hit by the bad shock ($s_1$), reflecting the excess demand for tradable goods in this country. Why is Foreign not offering tradable goods at a constant price? That would require Foreign consumers to shift consumption to the non-tradable sector but, although hit by a positive productivity shock, non-tradable goods are not cheap enough to induce such a shift in consumption. As a result, at a constant price, exports are not sufficient to provide full insurance to the more productive country. A larger number of multinationals in Foreign can then improve risk sharing. An increase in the number of firms results in cheaper non-tradable goods and larger price-elasticity with respect to the shock. In other words, the shift of consumption towards non-tradable goods is larger when the number of firms in the non-tradable sector increases. Then, if $Z_m > Z_m^*$, the gap in the price of Arrow-Debreu securities across states in an economy with MP (denoted by subscript MP) is smaller than in an economy with a complete set of securities but no reallocation of production (denoted by subscript C):

$$0 < q^{MP}(s_1) - q^{MP}(s_2) < q^{C}(s_1) - q^{C}(s_2).$$

Such a gap in the price of Arrow-Debreu securities across states enhances the value of Home MP (and reduces the value of Foreign MP). As analyzed in Section 2.2, with elastic demand
functions, MP profits co-move with host country risk. Consequently, MP profits of multinationals from Home are higher when world output is relatively scarce and Arrow-Debreu prices are high. The diagrams in Figure (1) illustrate this mechanism: net MP profits are higher when the host country is hit by a positive shock and the home country by a negative shock. If the two countries were equal, the steady state Arrow-Debreu prices would be constant across states (normalized to be equal to the probability of each state). However, once countries differ in their average productivity, Arrow-Debreu prices are higher in the state where the large country is hit by the negative shock. As a result, when MP profits are weighted by the corresponding Arrow-Debreu price, net MP profits of firms from the larger country are more valuable.

![Figure 1: MP Profits and Arrow-Debreu Prices](image)

As we stated above, the reallocation of production to the less productive economy improves the scope for risk sharing, giving more incentives to firms from the more productive to open affiliates abroad. Consider the case of a mean preserving spread over $A$ and $A^*$, which increases the value of insurance. From Proposition 1, the elasticity of Arrow-Debreu prices to a positive perturbation of $A$ (and a symmetric negative shock on $A^*$) is negative: $\xi_{qA} < 0$. Hence, an increase in the size of the shock widens the difference in the price of Arrow-Debreu securities.

$^{13}$See Appendix for the derivation of $\xi_{qA}$. 

18
across states: $q'(s_1) - q'(s_2) > q(s_1) - q(s_2)$, where $'$ reflects variables after the mean preserving spread. In other words, a mean preserving spread amplifies the difference in marginal utilities across states and therefore increases the value of insurance. As expected, the amount of Home firms doing MP increases, $[1 - G(z')] > [1 - G(z)]$, while the amount of MP by Foreign firms decreases, $[1 - G(z^*')] > [1 - G(z^*)]$. As a result, Home net discounted MP profits increase:

$$\sum_{s \in S} q'(s) \left( \Pi'_m(s) - \Pi'_m(s) \right) > \sum_{s \in S} q(s) \left( \Pi_m(s) - \Pi'_m(s) \right)$$

The counterpart of an increase in the net value of MP is a reduction in the position of Arrow-Debreu securities. Home’s demand for insurance is increasingly satisfied with profits from MP, while Foreign consumers rely more on Arrow-Debreu securities. Indeed, from the intertemporal budget constraint for Home, (12), and the initial wealth (14), the position on Arrow-Debreu securities $\sum_{s \in S} q(s) B(s)$ is given by the demand for resources to finance MP. A mean preserving spread increases both the number of firms from the Home country doing MP ($z' < z$) and the price of MP activities ($p'_f > p_f$) as insurance is more valuable when risk is larger. Then, Home’s position of Arrow-Debreu securities is lower in a world with higher risk:

$$\sum_{s \in S} q'(s) B'(s) - \sum_{s \in S} q(s) B(s) = p'_f \left[ Y_0 - (1 - G(z')) f^* \right] - p_f \left[ Y_0 - (1 - G(z)) f^* \right] < 0$$

Summarizing, a mean preserving spread over $A$ and $A^*$ increases the amount of MP done by

---

14 Notice that a mean preserving spread implies $\pi'_m(s_1) - \pi'_m(s_2) > \pi_m(s_1) - \pi_m(s_2)$, which combined with the widening in the gap in Arrow-Debreu prices across states results in an increase in the discounted MP profits of Home firms (and the opposite for Foreign’s). That is, for all $z : \sum_{s \in S} q'(s) \pi'(z, s) > \sum_{s \in S} q(s) \pi'(z, s) > \sum_{s \in S} q(s) \pi(z, s)$.

Then, for the marginal firms in Home and Foreign, $z$ and $z^*$, the following inequalities are satisfied (from MP entry condition):

$$\frac{\sum_{s \in S} q'(s) \pi'(z, s)}{f'} > \frac{\sum_{s \in S} q(s) \pi(z, s)}{f} = \frac{\sum_{s \in S} q(s) \pi(z^*, s)}{f} > \frac{\sum_{s \in S} q'(s) \pi'(z^*, s)}{f}.$$ 

which implies that a mean preserving spread lowers the cut-off level for Home firms ($z' < z$) and rises it for Foreign firms ($z'' > z^*$).
the most productive country and reduces its net Arrow-Debreu position. It is easy to see that Home’s overall net asset position is unambiguously improved when net FDI is measured as the discounted flow of net profits. Indeed, as the value of doing MP increases with \( z \) (from (24)), and from the MP entry condition (25 and 26), the value of MP is equal to the entry cost only for the marginal firm with productivity \( \bar{z} \), but positive for all other firms with \( z > \bar{z} \). Therefore, aggregate MP profits net of entry cost increase with the number of multinationals.

Concluding, MP has a role in international risk sharing in a world with asymmetric countries even when a complete set of financial assets: reallocation of production affects the relative size of the economies and the pattern of risk. In particular, risk gives firms from the large country more incentives to do MP and consequently: 1) brings countries closer together in terms of production size; 2) improves the scope for diversification across countries; and 3) reduces the differences between Arrow-Debreu prices across states. These results are larger when diversification of risk is more valuable, that is, the higher is the volatility of country-specific productivity shocks (or the higher the risk aversion).

This mechanism stems from four crucial assumptions of the model: (i) countries are asymmetric; (ii) demand functions for non-tradable goods are elastic; (iii) country-specific productivity shocks hit non-tradable relative to tradable sectors; and (iv) affiliates abroad bear the productivity shock specific to the host country.

3 Quantitative Analysis

To quantify the impact of risk on reallocation of production and the international position of countries, we calibrate the model to business cycle and external account moments of the US economy and an aggregate of developed and developing countries (ROW).\(^{15}\) We decompose the

\(^{15}\)Argentina, Austria, Australia, Belgium, Brazil, Canada, Chile, China, Colombia, Denmark, Egypt, Finland, France, Germany, Greece, Hong Kong, India, Indonesia, Ireland, Italy, Israel, Japan, Korea, Mexico, Malaysia,
period 1960-2005 into two sub-periods with substantially different GDP volatility. We calibrate the model to moments for the sub-period 1960-1984, and quantify the mechanism proposed in the paper by doing the following comparative statics exercise. In particular, we first compute the US Balance of Payment accounts implied by the 1960-1984 calibrated model. Second, we re-compute US Balance of Payment accounts using a different calibrated version of the model. In particular, we keep all parameters at the value implied by the 1960-1984 calibration except for the shocks’ volatilities that we recalibrate to the the US and ROW GDP volatilities observed in the sub-period 1985-2005. Moreover, we use as initial wealth for this second sub-period, the one implied by the 1960-1984 calibration of the model.

We find that in the sub-period 1960-1984, when world volatility is larger, world financial prices follow ROW risk. In contrast, in the sub-period 1985-2005, characterized by a lower volatility, world financial prices follow US risk. This means that Arrow-Debreu securities are more expensive when ROW is hit by a negative in the first sub-period, while they are more expensive when US is hit by a negative shock in the second sub-period. As a result, US incentives to do MP increase substantially in the latter sub-period: its Direct Investment position increases while its net international position in other assets is reduced.

3.1 Calibration

The second period in the model can be interpreted as the infinite future, with shocks following a stationary Markov chain and future consumption discounted at the rate β. In particular, we assume that country-specific shocks, A and A* follow a two-state Markov chain, symmetric and

Morocco, Netherlands, New Zealand, Norway, Pakistan, Peru, Philippines, Portugal, Singapore, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom, South Africa.
We impose the following restrictions on the transition probabilities for each country:

\[ \text{All variables are averages across states using the stationary unconditional probabilities.} \]

The problem presented in Section 2 can be re-written recursively in the following way when country shocks follow a stationary Markov chain. The (per-state) budget constraint is:

\[ P(s)C(s) + \int_{z \in Z} \theta'(z)Q(z, s)dG(z) + \sum_{s' \in S} q(s'|s)B(s') \]
\[ = LW + B(s) + \int_{z \in Z} \theta(z)[\pi(z, s) + Q(z, s)]dG(z) \]

where \( B(s') \) corresponds to a state-contingent one-period Arrow-Debreu security, and \( q(s'|s) \) is its price conditional on the realization of \( s \), given by the following Euler equation:

\[ q(s'|s) = \frac{q(s')}{q(s)} = \beta \frac{u'(C(s'))}{u'(C(s))} \frac{P(s)}{P(s')} \text{Pr}(s'|s), \]

and \( Q(z, s) \) corresponds to the market price of a firm with productivity \( z \):

\[ Q(z, s) = \sum_{s' \in S} q(s'|s)[\pi(z, s') + Q(z, s')] \]
Similarly, the value of doing MP for a firm with productivity $z$ is given by:

$$Q_{MP}(z, s) = \sum_{s' \in S} q(s'|s)[\pi_m(z, s') + Q_{MP}(z, s')]$$

Then, Arrow-Debreu securities can be reinterpreted as a portfolio position, denoted by $\hat{B}(s)$, and its (stochastic) rate of return $R(s'|s)$ can be computed accordingly:

$$\hat{B}(s) = \sum_{s' \in S} q(s'|s)B(s')$$
$$R(s'|s) = \frac{B(s') - \hat{B}(s)}{\hat{B}(s)}.$$  

We interpret the income from this portfolio position as income from assets other than Direct Investment, and refer to it as “Other Assets”.

We assume that the firm productivity parameter $z$ is drawn from a Pareto distribution in each country:

$$G(z) = 1 - \left(\frac{z}{z_{\text{min}}}\right)^{-\gamma}$$
$$G^*(z) = 1 - \left(\frac{z}{z^*_{\text{min}}}\right)^{-\gamma^*}.$$  

We calibrate the model parameters shown in Table 3 by targeting the moments in Table 1, for the sub-period 1960-1984. This table also shows the moments for the sub-period 1985-2005 against which we recalibrate shock volatilities and transition probabilities.

The parameters in Table 2 are not included in the calibration procedure, and are taken from the literature. Further, we normalize the following parameters: $\bar{A} = L = f^* = W = W^* = z_{\text{min}} = z^*_{\text{min}} = 1$, and assume that the initial wealth in the sub-period 1960-1984 is the same for
<table>
<thead>
<tr>
<th></th>
<th>1960-1984</th>
<th>1985-2005</th>
<th>Data source†</th>
</tr>
</thead>
<tbody>
<tr>
<td>US net exports (% of GDP)</td>
<td>-0.5%</td>
<td></td>
<td>BEA</td>
</tr>
<tr>
<td>ratio GDP US to GDP ROW</td>
<td>0.46</td>
<td></td>
<td>IFS (IMF)</td>
</tr>
<tr>
<td>Std. Dev. GDP US</td>
<td>2.9%</td>
<td>1.7%</td>
<td>Penn World Tables</td>
</tr>
<tr>
<td>Std. Dev. GDP ROW</td>
<td>2.85%</td>
<td>1.1%</td>
<td>Penn World Tables</td>
</tr>
<tr>
<td>Autocorrelation US GDP</td>
<td>0.64</td>
<td>0.84</td>
<td>Penn World Tables</td>
</tr>
<tr>
<td>Autocorrelation ROW GDP</td>
<td>0.78</td>
<td>0.67</td>
<td>Penn World Tables</td>
</tr>
<tr>
<td>Net factor income from other assets</td>
<td>0.02%</td>
<td></td>
<td>BEA</td>
</tr>
<tr>
<td>ROW MP sales in US (% of sales‡)</td>
<td>5%</td>
<td></td>
<td>BEA</td>
</tr>
<tr>
<td>ratio of US to ROW MP Profits</td>
<td>7.8</td>
<td></td>
<td>BEA</td>
</tr>
</tbody>
</table>

†: (i) Std. Dev. and autocorrelations refer to HP filtered (log of) p.c. GDP; ROW’s p.c. GDP is an average weighted by population; (ii) BEA data start in 1966 for BoP flows, 1976 for BoP stocks, and 1980 for multinational activity; (iii) IFS (IMF) data for total GDP start in 1980.
‡: we approximate total US sales by 3 times US GDP.

Table 1: Data Moments.

US and ROW, \( Y_0 = Y_0^* \).

Table 3 shows the parameters calibrated to the business cycle and external account moments in Table 1, for US and ROW, for the period 1960-1984. Alternatively, we calibrate the model to the same moments as in Table 1 but we use the ratio of US to ROW MP profits of 5.5, as reported by McGrattan and Prescott (2007). [Results of this alternative calibration are reported in the Appendix.]

Table 4 shows the model implied values for some moments, at the calibrated parameters,
3.2 Quantitative Exercise

We perform a comparative static exercise in order to quantify the importance of the role of MP in cross-country risk sharing. The exercise presented here measures the impact of a change in world volatility on the reallocation of production and the international portfolio composition. As shown in Table 1, there was a large decrease in world risk between the two sub-periods considered: GDP volatility for the US decreased from 2.9% to 1.7%, and for ROW, from 2.85% to 1.1%. Our exercise compares Arrow-Debreu prices and Balance of Payment accounts in two steady states: one consistent with the observed US and ROW volatilities for 1960-1984, and another with the ones observed for 1985-2005.

We proceed by re-calibrating the shock volatilities, $\Delta$ and $\Delta^*$ from equation (28) and (29), and transition probabilities, $Pr(s' = A_H | s = A_H)$ and $Pr(s' = A^*_H | s = A^*_H)$, to match the US and ROW GDP volatility, as well as the GDP autocorrelations, for the sub-period 1985-2005. The re-calibrated parameters are presented in Table 5. Additionally, we use the net asset position

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.98</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.7</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2</td>
</tr>
<tr>
<td>$\gamma^*$</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2: Parameters from literature.

and the actual data for the sub-period 1960-1984.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A/A^*$</td>
<td>11.5</td>
<td>US relative (mean) shock</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>0.42</td>
<td>volatility of US shock</td>
</tr>
<tr>
<td>$\Delta^*$</td>
<td>0.92</td>
<td>volatility of ROW shock</td>
</tr>
<tr>
<td>$Pr(A_H/A_H)$</td>
<td>0.82</td>
<td>US transition probability</td>
</tr>
<tr>
<td>$Pr^*(A_H/A_H)$</td>
<td>0.89</td>
<td>ROW transition probability</td>
</tr>
<tr>
<td>$L/L^*$</td>
<td>0.39</td>
<td>US relative size</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2</td>
<td>US productivity heterogeneity</td>
</tr>
<tr>
<td>$Y_0 = Y_0^*$</td>
<td>0.02</td>
<td>initial wealth</td>
</tr>
<tr>
<td>$f/f^*$</td>
<td>0.66</td>
<td>US fixed cost</td>
</tr>
<tr>
<td>$Z_m/Z_{m+d}$</td>
<td>0.29</td>
<td>US MP sales in ROW (% of ROW sales)</td>
</tr>
</tbody>
</table>

Table 3: Calibrated Parameters. 1960-1984

implied by the calibrated model for the period 1960-1984 as initial wealth for 1985-2005. The net asset position involves the the number of US and ROW multinational firms and the net position in Arrow-Debreu securities ("Other assets"), as reported in Table 7. The rest of the model parameters are left at the values reported in Table 3.

The Arrow-Debreu prices implied by the model calibrations are reported in Table 6. The first column reports the Arrow-Debreu prices implied by the model calibrated to 1960-1984 risk, and the second column the ones implied by the model calibrated to 1985-2005 risk.

Notice from Tables 3 and 5 that US is more productive than ROW (measured as $\overline{A}$) but
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta )</td>
<td>0.28</td>
<td>volatility of US shock</td>
</tr>
<tr>
<td>( \Delta^* )</td>
<td>0.35</td>
<td>volatility of ROW shock</td>
</tr>
<tr>
<td>( \Pr(A_H/A_H) )</td>
<td>0.92</td>
<td>US transition probability</td>
</tr>
<tr>
<td>( \Pr^*(A_H/A_H) )</td>
<td>0.83</td>
<td>ROW transition probability</td>
</tr>
</tbody>
</table>

Table 5: Re-calibrated Parameters. 1985-2005.

The volatility of the shock is lower in both calibrations (\( \Delta < \Delta^* \)). These two features have opposite implications for the relative impact of US and ROW shocks on Arrow-Debreu prices. In a world with high volatilities for both US and ROW, the volatility effect dominates and ROW has larger impact on Arrow-Debreu prices than US. The opposite occurs in the low volatility steady state. Focussing in the two states most relevant from a risk sharing point of view, \( s_1 \)
and $s_2$, in the steady state 1960-1984, Arrow-Debreu prices are larger when the bad shock affects ROW (and a positive shock affects US): $q^{60-84}(s_1) < q^{60-84}(s_2)$. In the lower volatility world (1984-2005), shocks affecting the US have larger impact on world financial prices; Arrow-Debreu prices are larger when the bad shock hits the US (and a positive shock affects ROW): $q^{85-05}(s_1) > q^{85-05}(s_2)$. The reduction in overall volatility also manifests in the narrowing of the Arrow-Debreu price gap across the four states, relative to the one implied by 1966-1984 volatilities (column 1). Indeed, we compute a two-fold decrease in the gap between Arrow-Debreu prices in the worst state ($s_3$) and the best state ($s_4$).

The effect of such a change in the relative impact of US and ROW shocks on Arrow-Debreu prices has an unambiguous prediction in terms of production reallocation. In the sub-period 1960-1984, ROW had a larger impact on international financial assets. Hence, reallocation of production from ROW to US improves the scope for risk sharing. In contrast, when world risk decreases, world financial assets follow US risk and, as a result, the scope for risk diversification improves with reallocation of production from US to ROW.

The values for the US external accounts for both steady states with high and low volatility
are presented in Table 7, column 1 and 2. We decompose the effect of reduction in volatility— and
the consequent change in Arrow-Debreu prices— into two: first, column 2 presents magnitudes
that correspond to a world in which the production structure is fixed at the values for 1960-1984
(“exogenous MP”); and second, the magnitudes in column 1 account for the optimal reallocation
of production triggered by the new Arrow-Debreu prices (“endogenous MP”).

The first rows of Table 7 show net exports, net factor income and current account (as % of
GDP) for the US, in the two steady states. Notice that net exports are negative as they are
financed with net income from abroad. Indeed, deficits are higher in a world with endogenous
MP. Yet, the current account is positive to repay the initial debt incurred to finance MP entry
costs.

Regarding US net Direct Investment position (measured as risk-adjusted discounted flow of
profits), it more than doubles (as % of GDP) between the two steady states. Indeed, the fraction
of US firms doing MP increases in detriment of firms from ROW doing MP in the US, as shown
in the last two rows. Notice that the US Direct Investment position increases in the second
steady state even if the number of US multinational firms is kept fixed, from 25% to 43% of
GDP.

Finally, the US goes from having a net positive position in other assets of 1.1% of GDP to a
negative one of 7.8%: the increase in the number of firms doing MP is financed with more debt.
The main difference in the alternative calibration presented in the Appendix is the implication
for this Balance of Payment account: the steady state with high volatilities displays a -10% US
net position in other assets, and reaches -17% in the low volatility steady state. As a counterpart,
the fraction of US multinational firms is much higher, going from 7.4% to 8.2% of total US firms
between steady states.

Table 8 quantifies the importance of the mechanism in the model by decomposing the total
change in Direct Investment Positions (measured as the discounted flow of MP profits), for US
### Table 7: Two Steady States: US External Accounts.

<table>
<thead>
<tr>
<th></th>
<th>Current Account</th>
<th>Net exports</th>
<th>Net Factor Payments</th>
<th>Net Income Direct Investment</th>
<th>Net Income Other Assets</th>
<th>Net Other Assets</th>
<th>Net Direct Investment Position$^\dagger$</th>
<th>% of US MP firms</th>
<th>% of ROW MP Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>(as % of US GDP)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steady State Values</td>
<td>$\Delta_{60-84} = 0.42$</td>
<td>$\Delta_{65-05} = 0.28$</td>
<td>$\Delta_{60-84}^* = 0.92$</td>
<td>$\Delta_{65-05}^* = 0.35$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>endogenous MP</td>
<td>0.1%</td>
<td>-0.50%</td>
<td>0.6%</td>
<td>0.6%</td>
<td>0.1%</td>
<td>1.1%</td>
<td>25%</td>
<td>2.2%</td>
<td>3.4%</td>
</tr>
<tr>
<td>exogenous MP</td>
<td>0.06%</td>
<td>-0.80%</td>
<td>0.85%</td>
<td>1%</td>
<td>-0.18%</td>
<td>-7.8%</td>
<td>62%</td>
<td>2.7%</td>
<td>2.6%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| $^\dagger$: measured as discounted flow of MP profits, discounted by AD prices ($Q_M$ and $Q^*_M$ in the model).
row “reallocation” correspond to changes due to the endogenous reallocation of MP firms.\textsuperscript{16}

<table>
<thead>
<tr>
<th>Direct Investment Position\textsuperscript{†}</th>
<th>US</th>
<th>ROW</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>total change</td>
<td>37%</td>
<td>-15%</td>
<td>152%</td>
</tr>
<tr>
<td>from valuation</td>
<td>24%</td>
<td>3%</td>
<td>70%</td>
</tr>
<tr>
<td>from reallocation</td>
<td>13%</td>
<td>-18%</td>
<td>82%</td>
</tr>
</tbody>
</table>

| Net Exports\textsuperscript{‡}           |        |        |        |
| total change                               | 114%   |        |        |
| from valuation                             | 74%    |        |        |
| from reallocation                          | 40%    |        |        |

\textsuperscript{†}: measured as $\sum_s q(s)\Pi_m(s)$ for US, and $\sum_s q(s)\Pi'_m(s)$ for ROW.
\textsuperscript{‡}: measured as $\sum q(s)NX(s)$.

Table 8: Decomposition: Direct Investment Position and Net Exports.

As expected, US Direct Investment position increases by 37\% between the two steady states, while ROW’s decrease by -15\%. Notice that the endogenous reallocation of production accounts for more than one third of the total change in US discounted MP profits when world risk changes. This translates in US \textit{Net} Direct Investment Position increasing by 1.5 times between steady states; the endogenous reallocation of production implied by our mechanism explains more than half of this change. Moreover, this endogenous reallocation of firms accounts for more than one

\[ \sum_s \{ q^{s_5 - s_0} (s) \Pi^{s_5 - s_0}_m(s) - q^{s_6 - s_4} (s) \Pi^{s_6 - s_4}_m(s) \} \]

\[ = \sum_s q^F(s) \Pi^F_m(s) - q^{s_6 - s_4} (s) \Pi^{s_6 - s_4}_m(s) + \sum_s q^{s_5 - s_0} (s) \Pi^{s_5 - s_0}_m(s) - q^F(s) \Pi^F_m(s) \]
third of the US net exports change: from -0.5% to -0.8% between steady states, with the model without endogenous MP reaching -0.6% of US GDP (Table 7).

As a counterpart, the US net position in “Other assets” deteriorates between the two steady states, from 1.1% to -7.8% of GDP (Table 7). The change due to the endogenous reallocation of production, represented by the increase in debt to finance new affiliates abroad, accounts for almost all the change in US net position in other assets: from 0.5% to -7.8% of US GDP (column 2 and 3 in Table 7). The change in Arrow-Debreu prices positively affects the valuation of the US net position in “Other asset”. International risk sharing implies $B(s_1) > B(s_2)$, that is, the US has relatively larger claims in the state where they are hit by a negative shock (and ROW is hit by a positive shock). Prices are higher in such state for the steady state with lower volatility –i.e. $q^{85-05}(s_1) - q^{85-05}(s_2) > 0 > q^{60-84}(s_1) - q^{60-84}(s_2)$ – which increases the valuation of US initial position. However, the new debt issued to finance new MP activities more than off-sets the positive valuation effect of US initial position. As a result, US ends up being a net debtor in the steady state corresponding to 1985-2005.

Performing the same decomposition as in Table 8 using the alternative calibrated set of parameters gives similar results: the endogenous reallocation of production accounts for almost two thirds of the total change in the US net Direct Investment position, and 40% of the increase in the trade deficit.

Summing up, the endogenous reallocation of production triggered by the change in volatility accounts for more than half of the total change in the US Net Direct Investment Position, mea-

\[
\sum_s \left( q^{85-05}(s) B^{85-05}(s) - q^{60-84}(s) B^{60-84}(s) \right) = \sum_s q^F(s) B^F(s) - q^{60-84}(s) B^{60-84}(s) + \sum_s q^{85-05}(s) B^{85-05}(s) - q^F(s) B^F(s)
\]
sured as risk-adjusted discounted flow of net MP profits. Similarly, the predictions in terms of US net position in “Other Assets” is fundamentally altered if we do not consider the endogenous reallocation of production. We find this result very suggestive: focusing on Direct Investment as a mere portfolio choice without considering the reallocation of production that such flow entails, potentially misses important effects on the composition of countries’ international portfolios.

4 Conclusions

This paper emphasizes the connection between production location and the pattern of international risk. In particular, the scope for international risk diversification is improved if production is reallocated towards economies with business cycles less correlated with the world risk process. Reallocation of production towards such economies may be triggered by a number of different factors, namely reduction of trade cost, improvements in the investment opportunities in those countries, or, as analyzed in this paper, ”Multinational Production”.

The main contribution of this paper is to uncover the dual role of MP as a production and portfolio flow in international risk sharing. Reallocation of production affects relative TFP across countries and alters the impact of country-specific shocks on global output. Moreover, as a counterpart, we find that risk affects the optimal location of firms. In particular, risk sharing considerations provide incentives to firms from the large country more incentives to do MP and consequently: 1) brings countries closer together in terms of production size; 2) improves the scope for diversification across countries; and 3) reduces the differences between Arrow-Debreu prices across states. These results are larger when diversification of risk is more valuable, that is, the higher is the variance of country-specific productivity shocks.

This mechanism stems from four crucial assumptions of the model: (i) countries are heterogeneous; (ii) demand functions for non-tradable goods are elastic; (iii) country-specific pro-
ductivity shocks hit non-tradable relative to tradable sectors; and (iv) affiliates abroad bear the productivity shock specific to the host country.

To quantify the impact of risk on reallocation of production and the portfolio decision of a country, we calibrate the model to business cycle and external account moments of the US economy and an aggregate of Rest of the World. Our exercise suggests that the endogenous reallocation of production triggered by a change in world volatility accounts for more than half of the total change in the US net Direct Investment Position measured as risk-adjusted discounted flow of net MP profits. Similarly, the predictions in terms of the net position in other assets is fundamentally altered if we do not consider the endogenous response in the reallocation of production. We find this result very suggestive: focusing on FDI as a mere portfolio choice without considering the reallocation of production that such a flow entails, is potentially missing important effects on international portfolio composition.

References


35


A Appendix: Proof of Proposition 1

Proof. We locally analyze the problem at \( A = A^* \) and evaluate a perfectly negatively correlated shock to \( A, A^* \) so that \( \xi_{A^*A} = -1 \). Define \( \xi_{xy} = \frac{\partial x}{\partial y} \). Differentiate (5) and (13) with respect to \( A \) to get the following expressions:

\[
\xi_{PA} = -\frac{P^{1-\rho} - 1}{P^{1-\rho}}
\]

\[
\frac{\partial \xi_{PA}}{\partial A} = -(1 - \rho) \frac{1 + \xi_{PA}}{A} \]

\[
\xi_{qA} = -\sigma (\xi_{CA} - \xi_{CA^*}) - \xi_{PA} = \sigma (\xi_{C^*A^*} - \xi_{C^*A}) + \xi_{P^*A^*}
\]

Rewrite the feasibility condition (27) as follows:

\[
L W \eta + L^* W^* \eta - (\eta + \xi_{P^*A^*}) X^* - (\eta + \xi_{PA}) X = 0
\]

Differentiate with respect to \( A \), the get:

\[
\xi_{qA} = \left( \sigma - 1 \right) \frac{\left( \eta + \xi_{PA} \right) \xi_{PA} X - \left( \eta + \xi_{P^*A^*} \right) \xi_{P^*A^*} X^*}{\left[ \left( \eta + \xi_{P^*A^*} \right) X^* + \left( \eta + \xi_{PA} \right) X \right]} + \sigma (\rho - 1) \frac{\left[ 1 + \xi_{PA} \right] \xi_{PA} X - \left[ 1 + \xi_{P^*A^*} \right] \xi_{P^*A^*} X^*}{\left[ \left( \eta + \xi_{P^*A^*} \right) X^* + \left( \eta + \xi_{PA} \right) X \right]}
\]

and

\[
sg \left( \frac{\partial \xi_{qA}}{\partial A} \bigg|_{A=A^*} \right) = \left\{ (\sigma - 1) + \sigma (\rho - 1) \right\} \left( 1 + 2 \xi_{PA} \right) + \left( \sigma - 1 \right) \left( \eta - 1 \right) \frac{\partial \xi_{PA}}{\partial A} A
\]

\[
+ \left\{ (\sigma - 1) + \sigma (\rho - 1) \right\} \left( 1 + \xi_{PA} \right) + \left( \sigma - 1 \right) \left( \eta - 1 \right) \xi_{PA} \left( \xi_{X} - \frac{\partial X^*}{\partial A} A \right)
\]

Define \( \phi = \frac{Z_d}{Z_d + Z_m} \). Evaluated at \( A = A^* \), the Current Account is balanced:

\[
WL \eta - (\eta + \phi \xi_{PA}) X - (1 - \phi^*) \xi_{P^*A^*} X^* = 0
\]

which, combined with the feasibility condition results in the following expression:

\[
\left( \xi_{X} \bigg|_{A=A^*} - \frac{\partial X^*}{\partial A} A \bigg|_{A=A^*} \right) = \left\{ \frac{1 - 2 \phi}{(\eta + \phi \xi_{PA}) - (1 - \phi) \xi_{P^*A^*} X^*} \right\} \frac{\partial \xi_{PA}}{\partial A} A
\]

If follows that \( \frac{\partial \xi_{qA}}{\partial A} \bigg|_{A=A^*} < 0 \) when \( \phi = 1 \) and \( \frac{(\eta - 1)}{(\rho - 1)} > \frac{\sigma}{(\sigma - 1)} \).

\( \square \)
Appendix: Alternative Calibration

This alternative calibration targets the same moments as in Table 1 but uses the ratio of US to ROW MP profits of 5.5 as reported by McGrattan and Prescott (2007). The calibrated parameters are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A/A^*$</td>
<td>8.7</td>
<td>US relative (mean) shock</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>0.41</td>
<td>volatility of US shock</td>
</tr>
<tr>
<td>$\Delta^*$</td>
<td>0.69</td>
<td>volatility of ROW shock</td>
</tr>
<tr>
<td>$\Pr(A_H/A_H)$</td>
<td>0.82</td>
<td>US transition probability</td>
</tr>
<tr>
<td>$\Pr^*(A_H/A_H)$</td>
<td>0.89</td>
<td>ROW transition probability</td>
</tr>
<tr>
<td>$L/L^*$</td>
<td>0.40</td>
<td>US relative size</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3.9</td>
<td>US productivity heterogeneity</td>
</tr>
<tr>
<td>$Y_0 = Y_0^*$</td>
<td>0.06</td>
<td>initial wealth</td>
</tr>
<tr>
<td>$f/f^*$</td>
<td>2.24</td>
<td>US fixed cost</td>
</tr>
<tr>
<td>$Z_m/Z_{m*}$</td>
<td>0.19</td>
<td>US MP sales in ROW (% of ROW sales)</td>
</tr>
</tbody>
</table>

Table 9: Calibrated Parameters. 1960-1984

<table>
<thead>
<tr>
<th></th>
<th>model</th>
<th>data</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Current Account (%GDP)</td>
<td>0.1%</td>
<td>0.3%</td>
</tr>
<tr>
<td>US Net Exports (% GDP)</td>
<td>-0.5%</td>
<td>-0.5%</td>
</tr>
<tr>
<td>US Net Factor Income (%GDP)</td>
<td>0.6%</td>
<td>0.8%</td>
</tr>
<tr>
<td>US Net Direct Investment Income (%GDP)</td>
<td>0.7%</td>
<td>0.8%</td>
</tr>
<tr>
<td>US Net Other Assets Income (%GDP)</td>
<td>-0.08%</td>
<td>0.02%</td>
</tr>
<tr>
<td>US Net Other Assets (%GDP)</td>
<td>-10%</td>
<td>1.8%</td>
</tr>
<tr>
<td>Correlation GDP(US,ROW)</td>
<td>0.41</td>
<td>0.68</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(as % of US GDP)</th>
<th>Steady State Values</th>
<th>[\Delta_{60-84} = 0.41]</th>
<th>[\Delta_{85-05}^* = 0.28]</th>
<th>[\Delta_{85-05}^* = 0.25]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current Account</td>
<td>0.1%</td>
<td>0.05%</td>
<td>0.06%</td>
</tr>
<tr>
<td></td>
<td>Net exports</td>
<td>-0.50%</td>
<td>-0.60%</td>
<td>-0.50%</td>
</tr>
<tr>
<td></td>
<td>Net Factor Payments</td>
<td>0.6%</td>
<td>0.65%</td>
<td>0.53%</td>
</tr>
<tr>
<td></td>
<td>Net Income Direct Investment</td>
<td>0.7%</td>
<td>1%</td>
<td>0.8%</td>
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<tr>
<td></td>
<td>Net Income Other Assets</td>
<td>-0.08%</td>
<td>-0.4%</td>
<td>-0.02%</td>
</tr>
<tr>
<td></td>
<td>Net Other Assets</td>
<td>-10%</td>
<td>-17%</td>
<td>-11%</td>
</tr>
<tr>
<td></td>
<td>Net Direct Investment Position[^\dagger]</td>
<td>37%</td>
<td>61%</td>
<td>46%</td>
</tr>
<tr>
<td></td>
<td>% of US MP firms</td>
<td>7.4%</td>
<td>8.2%</td>
<td>7.4%</td>
</tr>
<tr>
<td></td>
<td>% of ROW MP Firms</td>
<td>2%</td>
<td>1.65%</td>
<td>2%</td>
</tr>
</tbody>
</table>

\[^\dagger\]: measured as discounted flow of MP profits, discounted by AD prices \((Q_M, Q_M^*)\) in the model.

Table 11: Two Steady States: US External Accounts.