Profit Sharing: A Contracting Solution to Harness the Wisdom of the Crowd

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Wisdom of the crowd

Wisdom of the crowd (Surowiecki (2005))
- the collective opinion of a group of individuals
- often found to dominate the judgment of a single expert

Why does it exist? How prevalent is it?
- individual judgments often contain idiosyncratic noises
  ▶ averaging tends to cancel out these noises (law of large numbers)
- rooted in classic economic thoughts
- useful for modern settings (e.g. earnings forecast, crowdfunding)
  ▶ Da and Huang (2015), Brown and Davies (2015), Chemla and Tinn (2016), Xu (2016), etc.

This paper (assuming the existence of the wisdom of the crowd effect):
- how to best harness it? e.g. via “smart” contract design?
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An illustrative example

Two investors, Alice & Bob, participate in funding a risky, scalable project

- independently decide how much money to commit to the project
  - based on their optimal return–risk trade-off
- deep pocketed; identically risk averse

Both investors use their private information to guide investment decisions

- each investor’s private information contains idiosyncratic noises
- neither investor has access to the other’s private information

Q: How should Alice and Bob divide up any payoff from their investment?
How should Alice and Bob divide up the payoff?

The typical approach (common stock)
- rewards investors in proportion to their initial investment
- the more Alice has invested, the larger payoffs she will enjoy

But...is this really optimal?
- winner’s curse: risk-aversion limits investment amount

⇒ call for better risk sharing than common stocks

What if, Alice and Bob equally divide up any net payoff?
- i.e. profit sharing for harnessing the wisdom of the crowd!
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# Common stock vs. profit sharing

Assume that *net* return is realized as 10% ↑

## Common stock

<table>
<thead>
<tr>
<th></th>
<th>Inv.Amt</th>
<th>Shr.G.</th>
<th>Gross payoff</th>
<th>Individual payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>$200</td>
<td>2/3</td>
<td>((200 + 100) \times)</td>
<td>(330 \times 2/3 = 220)</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>$100</td>
<td>1/3</td>
<td>((1 + 10%) = 330)</td>
<td>(330 \times 1/3 = 110)</td>
</tr>
</tbody>
</table>

## Fifty-fifty profit sharing (assume no changes in investment)

<table>
<thead>
<tr>
<th></th>
<th>Shr.N.</th>
<th>Inv.Amt</th>
<th>Net payoff</th>
<th>Individual payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>1/2</td>
<td>$200</td>
<td>((200 + 100) \times)</td>
<td>(200 + 30 \times 1/2 = 215)</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>1/2</td>
<td>$100</td>
<td>10% = 30</td>
<td>(100 + 30 \times 1/2 = 115)</td>
</tr>
</tbody>
</table>

A bad deal for A? Optimal investment also changes under profit sharing...

- not necessarily a bad deal to get a smaller piece of a bigger pie!
Common stock vs. profit sharing: illustration

Common stock (i.e. no profit-sharing)

Fifty-fifty profit sharing (assume no investment change)
Common stock vs. profit sharing: illustration

Common stock
(i.e. no profit sharing)

Fifty-fifty profit sharing
(assume no investment change)

Assume that net return is realized as 10% ↑
A bad deal for A to go fifty-fifty?
Common stock vs. profit sharing: illustration

Common stock (i.e. no profit sharing)

A

220

B

110

Fifty-fifty profit sharing (investment optimally changes)

A

?

B

?

Not necessary...if A gets a smaller piece of a bigger pie!
Formal analysis of the illustrative example

Two deep-pocketed, identically risk averse investors \((i \in \{A, B\})\)
- maximize constant absolute risk aversion utility: \(u(W) = -e^{-\rho W}\)

The risky (scalable) prior with net return denoted as a random variable \(\tilde{r}\)
- investor \(i\)'s private signal \(s_i = r + \epsilon_i\)
  where \(r\) is the realization of \(\tilde{r}\), \(\epsilon_i \sim \mathcal{N}(0, \tau_i^{-1})\), \(\epsilon_i \perp \tilde{r}\), \(\epsilon_A \perp \epsilon_B\)
Optimal Investment under common stock

Investor $i$’s problem: invest $x'_i$ given $s_i$ s.t.

$$x'_i(s_i) = \arg\max_x E[-e^{-\rho \tilde{r} x} | s_i]$$

Assume $\tilde{r} \sim \mathcal{N}(\bar{r}, \tau^{-1}_r)$ for ease of exposition, RHS leads to

$$x'_i(s_i) = \arg\max_x -e^{-\rho E(\tilde{r} | s_i)x + \frac{1}{2} \text{Var}(\tilde{r} | s_i) \rho^2 x^2}$$

$$= \frac{1}{\rho} \left( \tau_r \bar{r} + \tau_i s_i \right)$$
If $A$ and $B$ could exchange private information before making investing decisions...

Then investor $i$ knew both $s_i$ and $s_{-i}$, and

$$x_i'(s_i, s_{-i}) = \arg\max_x \mathbb{E}[-e^{-\rho \tilde{r} x}|s_A, s_B]$$

RHS leads to

$$x_i'(s_A, s_B) = \arg\max_x -e^{-\rho \mathbb{E}(\tilde{r}|s_A, s_B)x} + \frac{1}{2} \text{Var}(\tilde{r}|s_A, s_B) \rho^2 x^2$$

$$= \frac{1}{\rho} (\tau_r \tilde{r} + \tau_A s_A + \tau_B s_B)$$

(full information benchmark)
Optimal Investment if $A$ and $B$ agree to share profits equally

Investor $i$’s problem: invest $x_i$ given $s_i$ s.t.

$$x_i(s_i) = \arg\max_x \mathbb{E}[-e^{-\frac{1}{2}\tilde{r}_i[x+\tilde{x}_i(s_i)]}|s_i]$$

∵ the RHS involves $i$’s belief of $\tilde{x}_i(s_i)$

• solution constitutes a Nash equilibrium

Definition

A Nash Equilibrium under an equal division of profits consists of two investment strategy functions $x_A(\cdot)$ and $x_B(\cdot)$ such that

$$x_i(s_i) = \arg\max_x \mathbb{E}[-e^{-\frac{1}{2}\tilde{r}_i[x+\tilde{x}_i(s_i)]}|s_i],$$

where $i \in \{A, B\}$ and $-i = \{A, B\}\backslash\{i\}$. 
Solving the Nash equilibrium

**Nash Equilibrium (from the Definition)**

\[
x_i(s_i) = \operatorname{argmax}_x \mathbb{E}[-e^{-\rho \frac{1}{2} \tilde{\rho} [x + \tilde{x}_{-i}(s_{-i})]} | s_i],
\]

(1)

Guess and verify a linear Nash equilibrium

\[
x_i(s_i) = \alpha + \beta_i s_i
\]

(1) \Rightarrow \alpha + \beta_i s_i = \operatorname{argmax}_x \mathbb{E}[e^{-\rho \frac{1}{2} \tilde{\rho} [x + \alpha + \beta_{-i} \tilde{s}_{-i}]} | s_i]

(2)

Both \(-\frac{1}{2} \rho \tilde{\rho}\) and \(x + \alpha + \beta_{-i} \tilde{s}_{-i}\) are normal r.v.-s conditional on \(s_i\)

\Rightarrow \text{expectation in the RHS of (2): m.g.f of a (general) } \chi^2\text{-r.v.}

- a closed-form expression exists
Profit sharing harnesses crowd wisdom

Under fifty-fifty profit sharing:

\[
\begin{align*}
    x_i &= \left( \tau_r \bar{r} + 2 \tau_i s_i \right) / \rho \\
    x_{-i} &= \left( \tau_r \bar{r} + 2 \tau_{-i} s_{-i} \right) / \rho
\end{align*}
\]

⇒ \( i \)'s payoff: \( r(\frac{x_i + x_{-i}}{2}) = \frac{r(\tau_r \bar{r} + \tau_A s_A + \tau_B s_B)}{\rho} \)

If \( A \) and \( B \) exchange private information before investing

\[
\begin{align*}
    x'_i(s_i, s_i) &= x'_{-i}(s_i, s_i) = \frac{(\tau_r \bar{r} + \tau_A s_A + \tau_B s_B)}{\rho} \\
    \Rightarrow \ i \)'s payoff: \( r x'_i(s_i, s_i) = \frac{r(\tau_r \bar{r} + \tau_A s_A + \tau_B s_B)}{\rho} \)
\]

Theorem

\( \forall \{r, s_A, s_B\} \), each investor's payoff under an equal division of profits always equals to that under a full information benchmark.
Why does profit sharing harness crowd wisdom?

Compare optimal investor behaviors:

- under common stock:

\[
\begin{align*}
  x'_i &= \frac{\tau \bar{r} + \tau_i s_i}{\rho} \\
  x'_{-i} &= \frac{\tau \bar{r} + \tau_{-i} s_{-i}}{\rho}
\end{align*}
\]

- under fifty-fifty profit sharing:

\[
\begin{align*}
  x_i &= \frac{\tau \bar{r} + 2\tau_i s_i}{\rho} \\
  x_{-i} &= \frac{\tau \bar{r} + 2\tau_{-i} s_{-i}}{\rho}
\end{align*}
\]
Consider $n$ investors each with risk-aversion $\rho_i$ and receiving $a_i$ of the profit

**Theorem (equilibrium existence and structure)**

Iff the pre-agreed profit ratio is proportional to risk tolerance, i.e.

$$a_i = \frac{1/\rho_i}{\sum_{i=1}^{n} 1/\rho_i},$$

a Nash equilibrium exists, under which each investor’s payoff is equal to what is under a full information benchmark.

Optimal sharing rule is easy to implement (only requires risk-aversions)

- individuals also have strict incentives to truthfully report their $\rho_i$-s
Implications for crowdfunding security design

In May 2016, the SEC further sanctioned investment crowdfunding
- under Title III of the Jumpstart Our Business Startups (JOBS) Act
- entrepreneurs directly solicit funding from a large number of investors
- contracts agreed to at the time of investment specify monetary payoffs

Q1: What type of contract is optimal? Still an open question.
- currently common stock, debt, or hybrids are all used in practice

Wisdom of the crowd: an acclaimed benefit of crowdfunding
- extensively discussed from the entrepreneur’s perspective:
  - aggregate investment provides useful information to the entrepreneur

Q2: Could the wisdom of the crowd also benefit investors themselves?
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Investment crowdfunding platforms...

- Nextseed
- StartEngine
- AngelList
- CrowdFunder
- Republic
- WeFunder
How robust is our main result? I

Empirically, only a small number of entrepreneurial ventures take off while most others fail – returns may be skewed...

⇒ our result is intact under skewed project returns

Theorem (Arbitrary distributions of project return)

∀ arbitrary distributions of project return \( \tilde{r} \) and an exponential family likelihood function of \( \tilde{r} \) given private signals \( s_i, i \in \{A, B\} \), profit sharing gives the same payoff for both investors as in a full-information benchmark.
How robust is our main result? II

Sensible to assume endowed private information in crowdfunding

- how will results change if private information has be costly acquired?
- a free-riding problem (Holmström (1982)) in information acquisition?
- e.g. assume constant marginal cost in acquiring signal precision

⇒ free-riding not large enough to cancel out the wisdom of the crowd

Theorem (Costly Information Acquisition)

\textit{With a constant marginal cost in acquiring private signal precision, investors strictly prefer more participants in profit sharing.}
How robust is our main result? III

Sensible to assume constant return to scale for crowdfunding projects
- how will results change for projects with (dis)economies of scale?
- e.g. assume total investment influences net return \( \tilde{r} - \lambda(x_1 + x_2) \)

\[ \Rightarrow \] the profit-sharing contract derived above is still first-best optimal

**Theorem (Projects with (dis)economy of scale)**

*The first-best allocation chosen by an omniscient and benevolent social planner could be sustained by a Nash equilibrium under profit sharing plus some cash transfers, even if the project features (dis)economy of scale.*

A Second Welfare Theorem under externality and asymmetric information?
A few further thoughts

Explain the structures of partnership firms?

Guide the design of Decentralized autonomous organizations (DAO)?
Or alternative financing such as initial coin offering (ICO)?
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Explain the structures of partnership firms?

- $t = 1$: determine shares
- $t = 2$: new business arrives, work & paid
- $t = 3$: divide residual earnings
- $t = 4$: 

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Or alternative financing such as initial coin offering (ICO)?
Brown, David C, and Shaun William Davies, 2015, Equity crowdfunding: Harnessing the wisdom of the crowd, Available at SSRN.


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