Technical Appendix: PRISM Model Documentation

Keith Sill

June 6, 2011

1 Model Structure

The FRBPHIL DSGE forecasting model (PRISM) is developed and maintained by the Real Time Data Research Center (RTDRC) and by the Research Department of the Federal Reserve Bank of Philadelphia. The model is medium-scale and features nominal and real frictions that include wage and price stickiness, habit formation, and capital adjustment costs. This section of the model documentation describes the DSGE model, which is essentially the model in Del Negro, Schorfheide, Smets, and Wouters (2007).

1.1 Final Goods Producers

There is a final good $Y_t$that is produced as a composite of a continuum of intermediate goods $Y_t(i)$using the technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{f,t}}} \right]^{1+\lambda_{f,t}} \tag{1}$$

with $\lambda_{f,t} \in (0, \infty)$ following the exogenous process:

$$\ln \lambda_{f,t} = (1 - \rho_{\lambda_f}) \ln \lambda_f + \rho_{\lambda_f} \ln \lambda_{f,t-1} + \sigma_{\lambda_f} ?_{\lambda,t} \tag{2}$$

The variable $\lambda_{f,t}$is the desired markup over marginal cost that intermediate goods producers would like to charge. From the first-order conditions for profit maximization and the zero-profit condition (final goods producers are perfectly competitive firms) the demand for intermediate goods is given by:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_{f,t}}{\lambda_{f,t}}} Y_t \tag{3}$$

with the composite good price given by:

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda_{f,t}}} di \right]^{-\lambda_{f,t}} \tag{4}$$

1.2 Intermediate Goods Producers

There is a continuum of intermediate goods indexed by i. They are produced using the technology:

$$Y_t(i) = \max\{Z_t^{1-\alpha} K_t(i)^{\alpha} L_t(i)^{1-\alpha} - Z_t \Phi, 0\} \tag{5}$$

where $Z_t$is exogenous technological progress that is assumed non-stationary. We define $z_t = \ln(Z_t/Z_{t-1})$and assume that it follows the process:

$$(z_t - \gamma) = \rho_z (z_{t-1} - \gamma) + \varepsilon_{z,t}.$$

Prices are assumed to be sticky and adjust following Calvo (1983). Each firm can readjust prices optimally with probability $1 - \zeta_p$in each period. Firms that are unable to reoptimize their prices $P_t(i)$adjust prices mechanically according to:

$$P_t(i) = (\pi_{t-1})^{\iota_p} (\pi_*)^{1-\iota_p} \tag{6}$$

where $\pi_t = P_t/P_{t-1}$and $\pi_*$is the steady state inflation rate of the final good. Those firms that re-optimize price choose a price level $\tilde{P}_t(i)$that maximizes the expected present discounted value profits in all states of nature in which the firm maintains that price in the future:

$$\begin{aligned} &max_{\tilde{P}_t(i)} \quad \Xi_r^p \left(\tilde{P}_t(i) - MC_t\right) Y_t(i) + \\ &E_t \sum_{s=1}^{\infty} \zeta_p^s \beta^s \Xi_{t+s}^p \left(\tilde{P}_t(i)(\Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}\right) - MC_{t+s}) Y_{t+s} \end{aligned} \tag{7}$$

subject to

$$Y_{t+s}(i) = \left(\frac{\tilde{P}_t(i) \left(\Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}\right)}{P_{t+s}} \right)^{-\frac{1+\lambda_{f,t}}{\lambda_{f,t}}} Y_{t+s}$$

where $\pi_t \equiv P_t/P_{t-1}$, $\beta^s \Xi_{t+s}^p$ is the household's discount factor and $MC_t$is the firm's marginal cost. Markets are assumed to be complete so all households face the same discount factor. All firms that can re-adjust price face an identical problem. We will consider only a symmetric equilibrium in which all adjusting firms choose the same price – which means that we can drop the $i$index. It then follows that the aggregate price level can be expressed as:

$$P_t = \left[(1 - \zeta_p) \tilde{P}^{-\frac{1}{\lambda_f}} + \zeta_p \left(\pi_{t-1}^{\iota_p} \pi_*^{1-\iota_p} P_{t-1} \right)^{-\frac{1}{\lambda_f}} \right]^{-\lambda_f}.$$

In the estimation, we shut down inflation indexation by setting $\iota_p = 0$.

1.3 Households

The objective function for household j is given by:

$$E_t \sum_{s=0}^{\infty} b_{t+s} \left[\ln(C_{t+s}(j) - hC_{t+s-1}(j)) - \frac{\varphi_{t+s}}{1+\nu_l} L_{t+s}(j)^{1+\nu_l} + \frac{\chi_{t+s}}{1-\nu_m} \left(\frac{M_{t+s}(j)}{Z_{t+s} P_{t+s}} \right)^{1-\nu_m} \right]$$

where $C_t(i)$is consumption, $L_t(i)$is labor supply, and $M_t(j)$is money holdings. Household preferences are subject to three shocks: an intertemporal shifter b_t, a labor supply shock φ_t, and a money demand shock χ_t. All preference shocks are assumed to follow an AR(1) process in logs. The household budget constraint, written in nominal terms, is given by:

$$P_{t+s}C_{t+s}(j) + P_{t+s}I_{t+s}(j) + B_{t+s}(j) \leq R_{t+s}B_{t+s-1}(j) + M_{t+s-1}(j) +$$
$$\Pi_{t+s} + W_{t+s}(j)L_{t+s}(j) + R^k_{t+s}u_{t+s}(j)\hat{K}_{t+s-1}(j) - P_{t+s}a(u_{t+s}(j))\hat{K}_{t+s-1}(j)$$

where $I_t(j)$is investment, $\hat{K}_t(j)$is capital holdings, $u_t(j)$is the rate of capital utilization, and $B_t(j)$is holdings of government bonds. The gross nominal interest rate paid on government bonds is $R_t$and $\Pi_t$is the per-capita profit the household gets from owning firms. Household labor is paid wage $W_t(j)$and households rent an "effective" amount of capital to firms $K_t(j) = u_t(j)\hat{K}_{t-1}(j)$. In return, they receive $R^k_t u_t(j)\hat{K}_{t-1}(j)$. Households pay a consumption cost associated with capital utilization given by $a(u_t(j))\hat{K}_{t-1}(j)$. Capital accumulation is governed by:

$$\hat{K}_t(j) = (1-\delta)\hat{K}_{t-1}(j) + \mu_t\left(1 - S\left(\frac{I_t(j)}{I_{t-1}(j)}\right)\right)I_t(j)$$

where $\delta$is the rate of depreciation, $S(\cdot)$is the cost of adjusting investment ($S' > 0$, $S'' > 0$), and $\mu_t$is a stochastic shock to the price of investment relative to consumption, assumed to follow an AR(1) process in logs.

1.4 The Labor Market

The labor market has labor packers that buy labor from households, combine it, and resell it to the intermediate goods producing firms. Labor used by the intermediate goods producers is a composite:

$$L_t = \left[\int_0^1 L_t(j)^{\frac{1}{1+\lambda_{w,t}}}\right]^{1+\lambda_{w,t}}$$

The labor packers maximize profits in a perfectly competitive environment, which leads to the labor demand:

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}}.$$

Combining labor demand with the zero-profit condition leads to the aggregate wage expression:

$$W_t = \left[\int_0^1 W_t(j)^{\frac{1}{\lambda_{w,t}}} di\right]^{\lambda_{w,t}}.$$

In the estimation, we fix $\lambda_{w,t} = \lambda_w \in (0, \infty)$. Households have market power, but wage adjustment is subject to a rigidity as in Calvo (1983). Each period, a fraction $1 - \zeta_w$of households re-optimize their wage. For those that are unable to re-optimize, $W_t(j)$adjusts as a geometric average of the steady state rate increase in wages and last period's productivity times last period's inflation. For those households that can re-optimize, the problem is to choose a wage $\tilde{W}_t(j)$that maximizes utility in all states of nature in which the household wage is to be held at its chosen value:

$$max_{\tilde{W}_t(j)} E_t \sum_{s=0}^{\infty} (\zeta_w \beta)^s b_{t+s} \left[-\frac{\varphi_{t+s}}{1+\nu_l} L_{t+s}(j)^{1+\nu_l} + \dots \right]$$

subject to

$$W_{t+s}(j) = \left(\Pi_{l=1}^{s} (\pi_*)^{1-\iota_w} (\pi_{t+l-1} e^{z^*_{t+l-1}})^{\iota_w} \right) \tilde{W}_t(j)$$

for $s = 1, \dots, \infty$ as well as to the household budget constraint and the labor demand condition. In the estimation, we shut down nominal wage indexation by setting $\iota_w = 0$.

1.5 Government Policies

The government consists of a fiscal authority and a monetary authority. The monetary authority sets the nominal interest rate according to the feedback rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_*} \right)^{\psi_R} \left(\frac{Y_t}{Y} \right)^{\psi_Y} \right]^{1-\rho_R} \epsilon_{R,t}.$$

The fiscal authority balances its budget by issuing short-term bonds. Government spending is exogenous and given by:

$$G_t = (1 - 1/g_t) Y_t$$

where the government spending shock $g_t$is assumed to follow an AR(1) process.

1.6 Exogenous Processes

There are seven exogenous shocks in the model. The follow the processes:

- Technology process. Let $z_t = \ln(Z_t / Z_{t-1})$

$$(z_t - \gamma) = \rho_z (z_{t-1} - \gamma) + \sigma_z \epsilon_{z,t}$$

- Preference for leisure:

$$\ln \phi_t = (1 - \rho_\phi) \ln \phi + \rho_\phi \ln \phi_{t-1} + \sigma_\phi \epsilon_{\phi,t}$$

- Money demand (this shock is shut down in the estimation and the model is not estimated using a monetary aggregate):

$$\ln \chi_t = (1-\rho_\chi)\ln \chi + \rho_\chi \ln \chi_{t-1} + \sigma_\chi \epsilon_{\chi,t}$$

- Price-markup shock:

$$\ln \lambda_t = (1-\rho_\lambda)\ln \lambda + \rho_\lambda \ln \lambda_{t-1} + \sigma_\lambda \epsilon_{\lambda,t}$$

- Capital adjustment cost (marginal efficiency of investment):

$$\ln \mu_t = (1-\rho_\mu)\ln \mu + \rho_\mu \ln \mu_{t-1} + \sigma_\mu \epsilon_{\mu,t}$$

- Intertemporal preference shifter:

$$\ln b_t = \rho_b \ln b_{t-1} + \sigma_b \epsilon_{b,t}$$

- Government spending shock:

$$\ln g_t = (1-\rho_g)\ln g + \rho_g \ln g_{t-1} + \sigma_g \epsilon_{g,t}$$

- Monetary policy shock:

$$\epsilon_{R,t}$$

1.7 Log-Linearized Model

Variables are detrended where appropriate and expressed as deviations from steady state.

- Detrending:

$$\begin{aligned}
&y_t = Y_t/Z_t,\ c_t = C_t/Z_t,\ i_t = I_t/Z_t,\ k_t = K_t/Z_t,\ \bar{k}_t = \bar{K}_t/Z_t,\\
&r_t^k = R_t^k/P_t,\ w_t = W_t/(P_t Z_t),\ \tilde{w}_t = \tilde{W}_t/W_t,\ \xi_t = \Xi_t Z_t,\\
&\xi_t^k = \Xi_t^k Z_t,\ z_t = \log(Z_t/Z_{t-1})
\end{aligned}$$

- Marginal cost:

$$mc_t = (1-\alpha)w_t + \alpha r_t^k. \tag{8}$$

- Phillips curve:

$$\pi_t = \beta\, \mathrm{E}_t\left[\pi_{t+1}\right] + \frac{(1-\zeta_p\beta)(1-\zeta_p)}{\zeta_p} mc_t + \frac{1}{\zeta_p}\lambda_{f,t}, \tag{9}$$

with normalization:

$$\lambda_{f,t} = [(1-\zeta_p\beta)(1-\zeta_p)\lambda_f/(1+\lambda_f)]\tilde{\lambda}_{f,t}$$

and $\lambda_f$is the steady state of $\tilde{\lambda}_{f,t}$.

- Capital-labor ratio:

$$k_t - L_t = w_t - r_t^k \tag{10}$$

- Marginal utility of consumption:

$$\begin{aligned}(e^\gamma - h\beta)(e^\gamma - h)\xi_t = -(e^{2\gamma} + \beta h^2)c_t + \beta h e^\gamma E_t[c_{t+1} + z_{t+1}] + \\ h e^\gamma (c_{t-1} - z_t) + e^\gamma (e^\gamma - h)\tilde{b}_t - \beta h(e^\gamma - h)\mathrm{E}_t[\tilde{b}_{t+1}]\end{aligned} \tag{11}$$

with the normalization:

$$\bar{b}_t = e^\gamma (e^\gamma - h)/(e^{2\gamma} + \beta h^2) b_t.$$

- Consumption euler equation:

$$\xi_t = \mathrm{E}_t[\xi_{t+1}] + R_t - \mathrm{E}_t[\pi_{t+1}] - \mathrm{E}_t[z_{t+1}]. \tag{12}$$

- Capital accumulation:

$$\begin{aligned}&k_t = u_t - z_t + \bar{k}_{t-1} \\ &\bar{k}_t = (2 - e^\gamma - \delta)[\bar{k}_{t-1} - a_t] + (e^\gamma + \delta - 1)[i_t + (1+\beta)S'' e^{2\gamma} \mu_t]\end{aligned} \tag{13}$$

- Investment:

$$i_t = \frac{1}{1+\beta}[i_{t-1} - z_t] + \frac{\beta}{1+\beta}\mathrm{E}_t[i_{t+1} + z_{t+1}] + \frac{1}{(1+\beta)S'' e^{2\gamma}}(\xi_t^k - \xi_t) + \mu_t \tag{14}$$

where $\xi_t^k$is the value of installed capital, evolving according to:

$$\xi_t^k - \xi_t = \beta e^{-\gamma}(1-\delta)\mathrm{E}_t[\xi_{t+1}^k - \xi_{t+1}] + \mathrm{E}_t[(1 - (1-\delta)\beta e^{-\gamma}) r_{t+1}^k - (R_t - \pi_{t+1})]$$

- Capital utilization:

$$u_t = \frac{r_*^k}{a''} r_t^k. \tag{15}$$

- Optimal real wage:

$$\begin{aligned}\tilde{w}_t = \zeta_w \beta E_t[\tilde{w}_{t+1} + \Delta w_{t+1} + \pi_{t+1} + z_{t+1}] + \\ \tfrac{1-\zeta_w\beta}{1+\nu_l(1+\lambda_w)/\lambda_w}(\nu_l L_t - w_t - \xi_t + \tilde{b}_t + \tfrac{1}{1-\zeta_w\beta}\varphi_t)\end{aligned} \tag{16}$$

- Real wage:

$$w_t = w_{t-1} - \pi_t - z_t + \frac{1-\zeta_w}{\zeta_w}\tilde{w}_t. \tag{17}$$

- Production function:

$$y_t = (1-\alpha)L_t + \alpha k_t \tag{18}$$

- Resource constraint:

$$y_t = (1+g_*)[\frac{c_*}{y_*}c_t + \frac{i_*}{y_*}(i_t + \frac{r_*^k}{e^\gamma - 1 + \delta}u_t)] + g_t \tag{19}$$

- Monetary policy rule:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)(\psi_1 \pi_t + \psi_2 y_t) + \sigma_R \epsilon_{R,t}. \tag{20}$$

2 Empirical Application

We use post-1983 U.S. data to estimate the DSGE model. We begin with a description of our data set and the prior distribution for the DSGE model parameters.

2.1 Data and Priors

Seven series are included in the vector of core variables $y_t$that is used for the estimation of the DSGE model: the growth rates of output, consumption, investment, and nominal wages, as well as the levels of hours worked, inflation, and the nominal interest rate. These series are obtained from Haver Analytics (Haver mnemonics are in italics). Real output is computed by dividing the nominal series (*GDP*) by population 16 years and older (*LN16N*) as well as the chained-price GDP deflator (*JGDP*). Consumption is defined as nominal personal consumption expenditures (*C*) less consumption of durables (*CD*). We divide by *LN16N* and deflate using *JGDP*. Investment is defined as *CD* plus nominal gross private domestic investment (*I*). It is similarly converted to real per-capita terms. We compute quarter-to-quarter growth rates as log difference of real per capita variables and multiply the growth rates by 100 to convert them into percentages.

Our measure of hours worked is computed by taking non-farm business sector hours of all persons (*LXNFH*), dividing it by *LN16N*, and then scaling to get mean quarterly average hours to about 257. We then take the log of the series multiplied by 100 so that all figures can be interpreted as percentage deviations from the mean. Nominal wages are computed by dividing total compensation of employees (*YCOMP*) by the product of *LN16N* and our measure of average hours. Inflation rates are defined as log differences of the core PCE deflator index (*JCXFE*) and converted into percentages. The nominal interest rate corresponds to the average effective federal funds rate (*FFED*) over the quarter and is annualized.

Our choice of prior distribution for the DSGE model parameters follows DSSW and the specification of what is called a "standard" prior in Del Negro and Schorfheide (2008). The prior is summarized in the first four columns of Table 1. To make this paper self-contained we briefly review some of the details of the prior elicitation. Priors for parameters that affect the steady state relationships, e.g., the capital share α in the Cobb-Douglas production function or the capital depreciation rate are chosen to be commensurable with pre-sample (1955 to 1983) averages in U.S. data. Priors for the parameters of the exogenous shock processes are chosen such that the implied variance and persistence of the endogenous model variables is broadly consistent with the corresponding pre-sample moments. Our prior for the Calvo parameters that control the degree of nominal rigidity are fairly agnostic and span values that imply fairly flexible as well as fairly rigid prices and wages. Our prior for the central bank's responses to inflation and output movements is roughly centered at Taylor's (1993) values. The prior for the interest rate smoothing parameter

ρ_R is almost uniform on the unit interval.

The 90% interval for the prior distribution on υ_l implies that the Frisch labor supply elasticity lies between 0.3 and 1.3, reflecting the micro-level estimates at the lower end, and the estimates of Kimball and Shapiro (2003) and Chang and Kim (2006) at the upper end. The density for the adjustment cost parameter S''spans values that Christiano, Eichenbaum, and Evans (2005) find when matching DSGE and vector autoregression (VAR) impulse response functions. The density for the habit persistence parameter $h$is centered at 0.7, which is the value used by Boldrin, Christiano, and Fisher (2001). These authors find that $h = 0.7$enhances the ability of a standard DSGE model to account for key asset market statistics. The density for a'' implies that in response to a 1% increase in the return to capital, utilization rates rise by 0.1 to 0.3%.

2.2 State Space Representation

The state space representation for the model estimation is given by:

$$S_t = T\ S_{t-1} + R\ e_t \tag{21}$$

with measurement equation:

$$\begin{bmatrix} \Delta \ln(y_t) \\ \Delta \ln(c_t) \\ \Delta \ln(I_t) \\ \ln(H_t) \\ \Delta \ln(W_t) \\ \pi_t \\ R_t \end{bmatrix} = D + Z * S_t \tag{22}$$

Note that we do not allow for measurement error in the estimation.

3 Parameter Estimates

Table 1: Prior and Posterior of DSGE Model Parameters (Part 1)

		Prior		Posterior	
Name	Density	Para (1)	Para (2)	Mean	90% Intv.
		Household			
h	$\mathcal{B}$	0.70	0.05	0.76	[0.71 , 0.81]
a''	$\mathcal{G}$	0.20	0.10	0.26	[0.10 , 0.43]
ν_l	$\mathcal{G}$	2.00	0.75	1.91	[1.07 , 2.69]
ζ_w	$\mathcal{B}$	0.60	0.20	0.74	[0.58 , 0.87]
$400 * (1/\beta - 1)$	$\mathcal{G}$	2.00	1.00	1.124	[0.37 , 1.86]
		Firms			
α	$\mathcal{B}$	0.33	0.10	0.16	[0.13 , 0.19]
ζ_p	$\mathcal{B}$	0.60	0.20	0.90	[0.89 , 0.92]
S''	$\mathcal{G}$	4.00	1.50	5.30	[3.22 , 7.25]
λ_f	$\mathcal{G}$	0.15	0.10	0.16	[0.01 , 0.31]
		Monetary Policy			
$400\pi^*$	$\mathcal{N}$	3.00	1.50	3.31	[2.60 , 4.17]
ψ_1	$\mathcal{G}$	1.50	0.40	2.25	[1.90 , 2.64]
ψ_2	$\mathcal{G}$	0.20	0.10	0.06	[0.04 , 0.08]
ρ_R	$\mathcal{B}$	0.50	0.20	0.81	[0.77 , 0.86]

Table 1: PRIOR AND POSTERIOR OF DSGE MODEL PARAMETERS (PART 2)

		Prior		Posterior	
Name	Density	Para (1)	Para (2)	Mean	90% Intv.
			Shocks		
400γ	$\mathcal{G}$	2.00	1.00	1.66	[1.17 , 2.13]
g^*	$\mathcal{G}$	0.30	0.10	0.28	[0.13 , 0.41]
ρ_a	$\mathcal{B}$	0.20	0.10	0.25	[0.14 , 0.36]
ρ_μ	$\mathcal{B}$	0.80	0.05	0.85	[0.80 , 0.90]
ρ_{λ_f}	$\mathcal{B}$	0.60	0.20	0.16	[0.07 , 0.26]
ρ_g	$\mathcal{B}$	0.80	0.05	0.96	[0.95 , 0.98]
ρ_b	$\mathcal{B}$	0.60	0.20	0.91	[0.87 , 0.95]
ρ_ϕ	$\mathcal{B}$	0.60	0.20	0.71	[0.56 , 0.91]
σ_a	$\mathcal{IG}$	0.75	2.00	0.63	[0.56 , 0.71]
σ_μ	$\mathcal{IG}$	0.75	2.00	0.39	[0.32 , 0.45]
σ_{λ_f}	$\mathcal{IG}$	0.75	2.00	0.17	[0.15 , 0.20]
σ_g	$\mathcal{IG}$	0.75	2.00	0.35	[0.31 , 0.39]
σ_b	$\mathcal{IG}$	0.75	2.00	0.50	[0.36 , 0.62]
σ_ϕ	$\mathcal{IG}$	4.00	2.00	9.08	[3.44 , 14.16]
σ_R	$\mathcal{IG}$	0.20	2.00	0.14	[0.12 , 0.16]

Notes: Para (1) and Para (2) list the means and the standard deviations for the Beta ($\mathcal{B}$), Gamma ($\mathcal{G}$), and Normal ($\mathcal{N}$) distributions; the upper and lower bound of the support for the Uniform ($\mathcal{U}$) distribution; s and ν for the Inverse Gamma ($\mathcal{IG}$) distribution, where $p_{\mathcal{IG}}(\sigma|\nu, s) \propto \sigma^{-(\nu+1)} e^{-\nu s^2/2\sigma^2}$. The joint prior distribution is obtained as a product of the marginal distributions tabulated in the table and truncating this product at the boundary of the determinacy region. Posterior summary statistics are computed based on the output of the posterior sampler. The following parameters are fixed: $\delta = 0.025$, $\lambda_w = 0.3$. Estimation sample: 1984:I to 2010:I.