Technical Details of the Aruoba Term Structure of Inflation Expectations (ATSIX)

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The ATSIX website contains a spreadsheet with output that is updated every month. The information in this spreadsheet allows the users to compute inflation expectations or real interest rates over any arbitrary horizon, including future rates, as long as the end of the horizon is within 120 months of the date of the update. In particular, users will want to use the four numbers reported in the "Factors" tab reading from the appropriate row that corresponds to the month as of which the forecast is to be made, which refers to the month t. The generic formula to compute the inflation expectations between month $t+\tau_1$ to $t+\tau_2$ with $120 > \tau_2 > \tau_1 > 0$ and $\tau_2 \geq 3$ is

$$\pi_{t+\tau_1 \to t+\tau_2} = L_t + \frac{e^{-\lambda\tau_1} - e^{-\lambda\tau_2}}{\lambda(\tau_2 - \tau_1)}(C_t - S_t) + \left(\frac{\tau_1 e^{-\lambda\tau_1} - \tau_2 e^{-\lambda\tau_2}}{\tau_2 - \tau_1}\right) C_t \tag{1}$$

where L_t, S_t, C_t and λ refer to "level", "slope", "curvature", and "lambda" in the spreadsheet.

The most obvious objects of interests are when $\tau_1 = 0$ and τ_2 varies between 3 and 120 – these are forecasts made as of period t for a horizon of τ_2 months. These are reported in the "InfExp" tab of the spreadsheet.

Also of interest will be what is known as five year, five-year forward (5f5) expectations, which is the expected inflation for the five-year period that starts at the end of the next five years from the point of forecast. The formula above can easily accomodate this with $\tau_1 = 60$ and $\tau_2 = 120$.

The "Real" tab of the spreadsheet contains real interest rates for horizons 3 to 120 months.[1]

[1]Note that the real rate computed here also contains inflation risk premium. See Aruoba (2016) for a discussion.

Some important notes:

1. In Aruoba (2016) the analysis is done by *smoothed* factors while ATSIX reports *filtered* factors in an effort to keep the historical values as real-time as possible.

2. The shortest horizon of survey forecasts used in the computation of ATSIX is three months and the longest is (approximately) ten years, and as a result we report ATSIX for horizons from 3 to 120 months.

3. All inflation rates are reported as annualized percentage rates. Also, as emphasized by Aruoba (2016), throughout the analysis assumes inflation between period t and $t+s$ is computed using the continuous compounding formula

$$\pi_{t\to t+s} \equiv 100 \times \frac{12}{s}\left[\log\left(P_{t+s}\right) - \log\left(P_t\right)\right] \tag{2}$$

   where P_t is the level of the CPI index in period t. As such inflation rate between two periods is equal to the average monthly inflation between the periods.

4. The model that underlies ATSIX has been initially estimated using data up to November 2015. In the September 2016 update, we included a number of model enhancements which required reestimation of the model's parameters. We also reestimated the deep historical inflation expectations and real interest rate term structures (first date to last date). In general, the historical data were not largely affected by the models enhancements, which included: adding additional Blue Chip and SPF forecasts at horizons not previously included and improving the way model treats some forecasts at horizons previously included. Given the major nature of the revision, we revised existing results. Subsequently we plan on reestimating the model roughly once every year. When we do, we will not revise existing results.

5. The spreadsheet contains results from the estimation done in July 2016 for forecasts made before and including July 2016. From August 2016 onwards the spreadsheet is updated in real time. In these updates unless it is a month where reestimation is done parameter estimates are kept same as the previous month. As such a month where reestimation is done can be identified by the changing value of "lambda" in the "Factors" tab of the spreadsheet.

6. Results from August 2016 and later are truly "real time" since no information that is realized after the month is used in the construction of the forecasts. For results prior

to August 2016, the only aspect of the analysis that violates real time is the fact that estimation of parameters uses the sample up to July 2016.

7. For computing the real interest rate yield curve, we use the nominal yield curve averaged over the first 15 calendar days of the month. This method stands in contrast to that Aruoba (2016), which averaged the daily nominal yield curve over the entire month, not the first 15 calendar days. The choice of a 15-day average is motivated by the ATSIX publication schedule: Because we publish the ATSIX monthly (after the 15-day average nominal yield curve is available but before we can compute a 30-day average), the 15-day average is both feasible in real time and serves to eliminate revisions to the previous observation on the estimated real yield curve, subject to the caveat in #6 above.

References

[1] Aruoba, S.B. (2016), "Term Structures of Inflation Expectations and Real Interest Rates," *Federal Reserve Bank of Philadelphia Working Paper 16-09/R.*