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AND MORTGAGE MARKETS
AND THE FORECLOSURE CRISIS**

Satyajit Chatterjee
Federal Reserve Bank of Philadelphia

Burcu Eyigungor
Federal Reserve Bank of Philadelphia

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A Quantitative Analysis of the U.S. Housing and Mortgage Markets and the Foreclosure Crisis*

Satyajit Chatterjee and Burcu Eyigungor
Federal Reserve Bank of Philadelphia

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Abstract

We present a model of long-duration collateralized debt with risk of default. Applied to the housing market, it can match the homeownership rate, the average foreclosure rate, and the lower tail of the distribution of home-equity ratios across homeowners prior to the recent crisis. We stress the role of favorable tax treatment of housing in matching these facts. We then use the model to account for the foreclosure crisis in terms of three shocks: overbuilding, financial frictions, and foreclosure delays. The financial friction shock accounts for much of the decline in house prices, while the foreclosure delays account for most of the rise in foreclosures. The scale of the foreclosure crisis might have been smaller if mortgage interest payments were not tax deductible. Temporarily higher inflation might have lowered the foreclosure rate as well.

Keywords: leverage, foreclosures, mortgage crisis

JEL Classifications: E21 E32 E44 G21 H24

*Corresponding author: Satyajit Chatterjee, Research Department, Federal Reserve Bank of Philadelphia, Ten Independence Mall, Philadelphia, PA 19106; 215-574-3861. E-mail: satyajit.chatterjee@phil.frb.org. We thank our editor Matthias Doepke, two anonymous referees, and Erhan Artuc for their very helpful comments. We are very grateful to Ronel Elul for help with data queries regarding home equity ratios at origination. We also thank seminar and conference participants at Bilgi University, the University of Maryland, the University of Melbourne, New York University, Reserve Bank of New Zealand, the Philadelphia Fed, Princeton University, the 2009 SED Meeting, the 2009 HULM Conference (Atlanta), the 2009 RSAI Conference and the 2010 NBER Summer Institute for their comments and thoughts. The views expressed in this paper are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or of the Federal Reserve System. This paper is available free of charge at www.philadelphiafed.org/research-and-data/publications/working-papers/.

1 Introduction

This study is motivated by the collapse in house prices and by rising defaults on mortgages in the United States during the recent financial crisis. It has two goals: first, to present a new model of long-duration collateralized debt obligations with default that can match key long-run features of the U.S. housing and mortgage markets, and second, to use the model to gain a quantitative understanding of the recent foreclosure crisis.

The main elements of the model are as follows. The economy is endowed with an exogenously given stock of rental and owner-occupied housing space. There is a continuum of infinitely lived individuals subject to uninsurable idiosyncratic shocks to earnings. People buy consumption goods and save in the form of risk-free savings accounts. An individual who is currently a renter can choose to purchase his housing space, offering the space as collateral in the mortgage market. The mortgage contract has a long duration, and borrowers freely choose their down payment. An individual who is currently a homeowner can choose to sell his house, default on the mortgage (if he has one), or simply keep his house. A renter can choose to continue to rent or purchase a house; a renter with a record of default does not have access to the mortgage market. There is a competitive intermediation sector that accepts savings from individuals and makes loans to borrowers at an interest rate that exactly reflects the borrower's probability of default. The intermediation sector also owns and operates the rental properties. Each period, the rental rates and the price of owner-occupied housing space are determined by equality of demand and supply in the two markets. All individuals pay income taxes as per the U.S. tax code, and homeowners pay property taxes as well. The model also features developers that play a role in the crisis.

We show that the model can be calibrated to match the average homeownership and foreclosure rates and the lower tail of the home equity distribution. The tax treatment of housing plays a key role in bringing the model close to reality. The exemption of implicit rental income from income taxes provides an important tax-saving motive for homeownership. The mortgage interest deduction offers incentives to take on leverage to purchase homes and helps account for the average level of foreclosures. Steady-state inflation, as well as the fact that homeowners steadily pay down their debt, helps account for the dispersed distribution of home equity seen in the data. The model

makes reasonable predictions regarding relevant data moments not targeted in the calibration.

To understand the foreclosure crisis, we chose three factors that seemed relevant a priori: an overbuilding of housing, a disruption in the flow of credit to the mortgage market, and delays in completing foreclosures. The first two are obvious choices.¹ Delays in completing foreclosures — which means that a defaulter does not have to vacate the house right after default — raises the value of default since the defaulter gets to live “rent free” for the duration of the delay (Ambrose, Buttimer, and Capone (1997)). Zhu and Pace (2011) show that anticipated foreclosure delays positively influenced the foreclosure rate during the crisis.

Incorporating these three factors into the model, we find that they can account for all of the 19 percent cumulative decline in prices over the crisis years and most (86 percent) of the 16 percent cumulative rise in foreclosures. We then use our model to assess the marginal contribution of each factor.

We find that the disruption to the flow of mortgage credit is key for accounting for the observed decline in house prices. In the absence of this disruption, house prices decline a little less than 6 percent. But surprisingly, there is no corresponding large reduction in the rise of the foreclosure rate: The foreclosure rate still increases a hefty 10.51 percent. Thus, the financial disruption accounts for about 69 percent of the observed decline in house prices but only 20 percent of the observed jump in foreclosures.

In contrast, the foreclosure processing delay, which allows a defaulter to live rent free for a year with some probability, is an important inducement to default. In its absence, the house price drop would still be about 19 percent, but the foreclosure rate would rise to only about 8 percent. Thus, foreclosure delays play no role in accounting for the drop in house prices but account for 37 percent of the observed rise in foreclosures.

The supply shock is important as well. In the absence of supply shock, house prices would decline 12.33 percent and foreclosure rate would be 7.38 percent. The overbuilding shock accounts for 35 percent of the observed drop in price and 40 percent of the rise in foreclosures, confirming

¹The construction boom that preceded the housing crash most likely involved some level of overbuilding, and the financial crisis adversely affected the functioning of the U.S. mortgage market.

the common notion that excessive homebuilding was an important element in the crisis.²

In addition to these findings, our model permits us to explore the implications of certain types of policy actions. We find that in the face of crisis, (temporarily) higher inflation leads to a lower foreclosure rate but almost the same drop in prices. We find that the shocks would have had a considerably smaller effect on foreclosures if the tax code did not encourage leverage.

There are two aspects in our study of the foreclosure crisis that are worth pointing out. First, the marginal contributions we report are contributions in the *accounting*, not causal, sense. This is because we treat the three shocks as independent when, in reality, they are most likely not so.³ Nevertheless, our accounting is valuable (we think) for giving us a sense of the importance of different channels that a more comprehensive theory of the crisis may draw upon.

Second, our study does not address why house prices rose before the crash. There are (at least) two distinct possibilities. One possibility is that the house prices had a “rational bubble” component that burst in 2006 (Barlevy and Fisher (2011)). Another possibility is a relaxation of household borrowing constraints that lenders thought was permanent (which led to the boom) but turned out not to be so (which led to the crash). This possibility has been explored in Boz and Mendoza (2014) and Favilukis, Ludvigson, and Nieuwerburgh (2013). Our paper relates to both possibilities. If the crash in house prices was due to the bursting of a rational bubble, that would explain the post-crash oversupply of housing (see, for instance, the discussion of house price bubbles in Blanchard and Watson (1982)). If the boom resulted from a relaxation of borrowing constraints erroneously perceived to be permanent, the financial shock in our model can then be interpreted as the unanticipated *reversal* of this relaxation. What we add relative to Boz and Mendoza and Favilukis, Ludvigson, and Nieuwerburgh is an understanding of the factors underlying default on mortgages during the crisis.

There are two quantitative-theoretic studies that account for certain long-run features of the U.S.

²The sum of the marginal contributions of the three shocks exceed the overall decline in prices and the overall rise in the foreclosure rate because of (nonlinear) interactions among the three shocks.

³For instance, the initial jump in foreclosures may have been triggered by the fall in house prices, resulting from overbuilding. Because of the concentration of risky mortgage lending among large banks and the subsequent run on these banks, the initial jump reduced the flow of credit to the mortgage market as a whole. The “credit crunch” forced steep drops in house prices and more foreclosures. The increased volume of foreclosures caused foreclosure delays that induced even more default. In this causal chain, the overbuilding shock causes the other two shocks.

housing and mortgage markets with the goal of gaining a better understanding of the foreclosure crisis. Garriga and Schlagenhauf (2009) account for the fraction of different types of mortgages, noting that subprime mortgages display a higher foreclosure rate than do prime mortgages, and analyze the impact on mortgage defaults of an unanticipated decrease in house prices resulting from a decline in construction costs.⁴ Corbae and Quintin (2015) consider an exogenous three-state Markov process for house prices calibrated to actual Case-Shiller home price index and allow two down payment options (0 percent and 20 percent) on long-maturity mortgages. The goal is to quantify the contribution of an endogenous rise in zero down payment (high leverage) mortgages to the foreclosure crisis.

Relative to these studies, our paper advances our understanding of the crisis in three ways. First, as previously stated, our goal is to understand factors underlying the decline in house prices as well as the rise in foreclosures. In contrast to both papers, rents and house prices in our model are determined by the equality of supply and demand. This allows us to take into account feedback between prices and foreclosures and helps us to understand the importance of the different factors driving the foreclosure crisis. Second, in contrast to both studies, we match the lower tail of the home equity distribution across households prior to the crisis since this distribution is a key determinant of the fraction of homeowners with negative (net) home equity following the house price decline. Finally, whether a negative home equity borrower defaults depends on the benefits of homeownership. In contrast to these two studies, we incorporate the benefits that flow from the preferential tax treatment of housing.⁵ This allows us to explore the effects of tax policy on housing and mortgage market outcomes during the crisis and in the long run.

We build on a growing quantitative-theoretic literature that addresses various aspects of the housing sector. In terms of modeling the housing sector, we follow (Gervais (2002)) in conceiving of the housing market as a market for homogeneous housing *space* (as opposed to houses) and in giving prominence to the preferential tax treatment of housing for understanding housing market outcomes.⁶ We go beyond Gervais (2002) (and a host of other studies) in allowing for the possibility

⁴In Garriga and Schlagenhauf’s model, the price of housing space is determined by the marginal cost of new construction, which is taken as technologically given.

⁵Corbae and Quintin assume that there is an “ownership premium” in preferences, while Garriga and Schlagenhauf assume that rental space depreciates faster than owner-occupied space. In our model, rental space also depreciates faster than owner-occupied space, and, in addition, there are tax benefits to owner occupancy.

⁶Gervais (2002) analyzed the distortions resulting from the special tax treatment of housing; namely, the failure

of default on mortgages.⁷ In terms of modeling the mortgage market, we follow Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) in assuming that each loan is competitively priced to reflect the objective probability of default on the loan (individualized or risk-based pricing). This approach is also taken in Jeske, Krueger, and Mitman (2013) and Guler (2014).⁸ We go beyond Jeske, Krueger, and Mitman in modeling mortgages as long-term contracts, wherein the obligation of the borrower to the lender diminishes over time and the borrower steadily accumulates equity in the house.⁹ We also advance the literature on consumer default by extending the long-maturity unsecured debt framework developed in Chatterjee and Eyigungor (2012) to an environment in which long-maturity debt is issued against collateral with value that may fluctuate over time. Luzzetti and Neumuller (2014) use this extension to study the interaction between bankruptcy reform and the mortgage crisis.

2 Environment

Time is discrete and indexed by $t = 0, 1, 2, \dots$. The economy comprises a household sector, a financial intermediation sector, and a developer sector. The economy has a given aggregate stock of housing space that can be owner-occupied, denoted H_O , and a given stock of housing space that can be rented by individuals, denoted H_R .

to tax the implicit rental income from owner-occupancy and the tax deductibility of mortgage interest income, in the context of a multigeneration- overlapping-generations model.

⁷Nakajima (2010) uses the same structure as Gervais to study the optimal capital income tax rate when residential capital is a significant portion of tangible capital and residential capital is treated preferentially in the tax code. Diaz and Luengo-Prado (2010) employ an infinite horizon framework to study the joint distribution of capital and housing stocks across the population. Chambers, Garriga, and Schlagenhauf (2009a) use a life-cycle structure similar to that of Gervais to study the role of demographics and mortgage innovation in the evolution of the homeownership rate since World War II. Chambers, Garriga, and Schlagenhauf (2009b) examine the aggregate consequences of different mortgage contracts. Rios-Rull and Sanchez-Marcos (2008), following on the earlier work of Ortalo-Magne and Rady (2006), model the housing sector as composed of different types of housing and study the migration of households from one type of housing to another. The role of housing investment in aggregate fluctuations has been analyzed by Davis and Heathcote (2005), Iacoviello and Pavan (2013), and Kiyotaki, Michaelides, and Nikolov (2011), among others. None of these papers has default on mortgages in equilibrium.

⁸Jeske, Krueger, and Mitman (2013) quantify the macroeconomic effects of the subsidy provided by the implicit federal guarantee of GSE debt in the context of an infinite-horizon economy. Guler (2014) examines the impact of better information on household default risk on loan-to-value ratios and interest rates in the mortgage market in the context of a life-cycle model. Both studies allow for default on mortgages.

⁹Jeske, Krueger, and Mitman model mortgages as one-period contracts that are “refinanced” each period.

2.1 Household Sector

There is a fixed continuum of individuals. Individuals derive utility from the consumptions of a homogeneous numeraire good and the service flow from housing space. Let $c(t)$ and $h(t)$ denote the two types of consumptions in period t . Then,

$$U(c, h) = \sum_{t=0}^{\infty} \beta^t u(c(t), h(t)), \quad 0 < \beta < 1, \quad (1)$$

where $u(c, h)$ satisfies standard assumptions.

Individuals independently draw an earnings level w according to a common finite-state Markov process with strictly positive support $W \subset \mathbb{R}_{++}$. The probability that $w(t+1) = w'$ given $w(t) = w$ is $F(w', w)$.

In period 0, individuals are endowed with some nonnegative level of financial wealth a and, potentially, some strictly positive level of owner-occupied housing space k . Individuals who have an endowment of housing space may have a mortgage against their house.

At any point, housing stock owned by an individual is subject to (random) depreciation at the rate $\delta \in (0, 1)$ with probability $\xi > 0$. Let δ_k denote the random variable that takes value δ with probability ξ and 1 with probability $(1 - \xi)$.

2.2 Financial Intermediaries

There is one representative risk-neutral financial intermediary that acts competitively. The intermediary owns the stock of rental housing space and rents it out to individuals. In addition, it offers interest-bearing deposits to individuals and makes mortgage loans to homeowners. It can borrow or lend funds in a world credit market at a given risk-free interest rate $r > 0$.

At any point, the rental housing stock in the hands of the intermediary depreciates at the (nonrandom) rate $\Delta \in (0, 1)$.

2.3 Developers

Developers are entities that may own a part of H_O in period 0. They can either sell their stock to individuals and/or hold it in inventory for future sale. Any housing in the hands of developers depreciates at the rate Δ . They, too, have access to the world credit market at the risk-free interest rate r . Once developers sell all their stock, they exit the economy.

2.4 Market Arrangement and Tax System

There are four markets in this economy. To properly account for the effect of inflation, we will denote the nominal price of the period t consumption good by $\Pi(t)$ so that the inflation rate between period t and $t + 1$ is $\pi(t + 1) = \Pi(t + 1)/\Pi(t) - 1$. The path of $\pi(t)$, $t \geq 0$, is exogenously given.

- There is a market for owner-occupied housing. In terms of the period t numeraire good, the price of one unit of housing space in period t is $p(t)$.
- There is a market for rental housing. In terms of the period t numeraire good, the price of one unit of housing space in period t is $z(t)$.
- There is a market for risk-free deposits that offers households the constant and exogenously given risk-free real interest rate r_f on deposits with taxable interest income. There is also a market for risk-free deposits that offers households a real return of r_e that is exempt from taxes. Individuals are restricted to holding a fraction $(1 - \omega)$ of their total deposits in tax-exempt form and the complementary fraction ω in taxable form.
- Finally, there is a market for mortgages in which an individual can borrow in *nominal* terms by offering his house as collateral. If an individual takes out a mortgage in period t , he agrees to make a sequence of nominal payments $\{X_{t+j}\}$, $j \geq 1$, starting in period $t + 1$. The size of the first payment, X , is chosen by the borrower but subsequent payments follow a geometrically declining path, with $X_{t+j} = \mu^{j-1}X$, $\mu < 1$. This structure is meant to mimic a mortgage contract with constant nominal payments for a fixed number of periods and zero

payments thereafter.¹⁰ In case of default, the lender gets ownership of the housing space offered as collateral.¹¹ In case of a sale, the lender receives

$$X_\tau \left(\frac{1}{1+i(\tau+1)} + \frac{\mu}{(1+i(\tau+1))(1+i(\tau+2))} + \dots \right),$$

where τ is the time of sale and $1+i(\tau+1) = (1+r_f)(1+\pi(\tau+1))$, where $\pi(\tau+1)$ is the inflation rate between period τ and $\tau+1$. This is simply the nominal present value of the remaining promised sequence of nominal payments discounted at the nominal risk-free rate facing the financial intermediary. The fact that $\mu < 1$ implies that the nominal value of the mortgagee's obligation declines over time. Because of the possibility of default, the period t price of a unit mortgage (namely, a mortgage that promises to pay the nominal sequence $\{1, \mu, \mu^2, \dots\}$) starting in period $t+1$ will depend on the characteristics of the individual taking out the mortgage. This will be described in more detail below.

Taxes are modeled after the U.S. tax system. Taxable income is computed in nominal terms, with deductions allowed for mortgage interest payments and property taxes. Let W be the nominal wage of an individual, A be the nominal value of beginning-of-period financial wealth (deposits), X be the nominal payment on the mortgage, K be the nominal value of the house (if the individual is a homeowner), and $Q(t)$ denote the present value in nominal terms of the nominal stream $\{1, \mu, \mu^2, \dots\}$ starting next period, discounted at the risk-free nominal interest rate. Then, the individual's taxable income I is given by:

$$I = \max\{0, W + \omega i(t)A - \max\{[1 - (1 - \mu)Q(t)]X + \rho K, S\}\} \quad (2)$$

Taxable income cannot be negative. If it is positive, it consists of labor earnings of the individual plus the interest income on taxable deposits (recall that a fraction $(1 - \omega)$ of deposits are tax exempt), less deductions. The deductions allowed are the maximum of the standard deduction S and the sum of itemized deductions. For homeowners, allowed deductions include property taxes,

¹⁰This structure eliminates the “time-to-termination” as an additional state variable in the pricing equation for mortgages. Defaultable bonds with geometrically declining coupon payments have been analyzed in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012).

¹¹We assume that when the lender gets ownership of the house following default, the borrower's obligation to the lender is extinguished and there is no recourse for the lender.

where ρ is the property tax rate, and interest payments on the mortgage (if any). To understand the interest payment term, observe that the value of debt bought back by an individual whose period t mortgage payment is X is $(X - \mu X)Q(t)$, so the portion of X that is interest payment is $[1 - (1 - \mu)Q(t)]X$.¹² The individual's nominal tax liability is then given by

$$G = \rho K + \int_0^I T(Y) dY, \quad (3)$$

where $T(\cdot)$ is the marginal federal tax rate and is weakly increasing in taxable income.

To express this tax liability in real terms, the following notation is used. We will denote $X/\Pi(t)$ by x , $S/\Pi(t)$ by s , and $A(t+1)/\Pi(t)$ by a' (which implies that a is $A(t)/\Pi(t-1)$). With these conventions, the real value of an individual's current taxable income, $I(t)/\Pi(t)$, is

$$\max\{0, w + \omega i(t)a/\pi(t) - \max[1 - (1 - \mu)q(t)/(1 + \pi(t+1))]x + \rho p(t)k', s\}, \quad (4)$$

where $q(t)$ is the present value of the real stream $\{1, \mu/(1+\pi(t+2)), \mu^2/(1+\pi(t+2))(1+\pi(t+3)), \dots\}$ starting next period discounted using the real risk-free interest rate r_f .¹³ The real value of the individual's tax liability is

$$g(w, a, x, k', t) = \rho p(t)k' + \int_0^{I(t)/\Pi(t)} \tau(y) dy, \quad (5)$$

where $\tau(\cdot)$ is the marginal tax rate when income is measured in terms of the current period numeraire good (we assume that nominal tax brackets move up with inflation one-for-one).

¹²If $\mu = 1$ (the mortgage is a perpetuity), then all of X is interest payment; if $\mu = 0$, the mortgage is a one-period bond and the interest payment is $X(t) \cdot [i(t+1)/(1+i(t+1))]$. Note that the interest deduction is based on the risk-free rate rather than on the implicit interest rate paid by the borrower. However, in our model, most borrowers choose to borrow at very close to the risk-free rate (i.e., they choose their down payment to reduce the default premium to almost zero), so this discrepancy is not very important.

¹³Note that $Q(t) = q(t)/(1 + \pi(t+1))$.

3 Decision Problems

3.1 Households

For a homeowner, the individual-level state variables are w, a, x, k , and δ_k . For a renter, the individual-level state variables are w and a and whether the renter is excluded from the mortgage market because of a previous default. For all individuals, the current and future values of all market prices, nominal interest rates, and inflation rates are aggregate state variables. In what follows, we summarize the path of aggregate state variables by the time index t . Denote the value function of a homeowner by $V_O(w, a, x, k, \delta_k, t)$, that of a renter who is not excluded from the mortgage market by $V_R(w, a, t)$, and that of a renter who is excluded by $V_R^D(w, a, t)$.

Consider the decision problem of a renter who has access to the mortgage market. If this individual chooses to purchase a home, he solves:

$$M_1(w, a, t) = \max_{c \geq 0, k' \geq 0, x' \geq 0, a' \geq 0} \{u(c, k') + \beta E_{(w', \delta'_k | w)} V_O(w', a', x', k', \delta'_k, t + 1)\}$$

$$c + g(w, a, 0, k', t) + a' + [1 + \chi_B]p(t)k' = w + a(1 + r) + q(w, a', x', k', t) \cdot x',$$

where $(1 + r) = \omega r_f + (1 - \omega)r_e$, χ_B is the proportional transactions cost of purchasing a house and $q(w, a', x', k', t)$ is the mortgage pricing function. Observe that payment on the chosen mortgage begins in the next period, so $x = 0$ in the tax liability function.

If the individual is excluded from the mortgage market due to a previous default but chooses to purchase a house, he solves:

$$M_1^D(w, a, t) = \max_{c \geq 0, k' \geq 0, a' \geq 0} \{u(c, k') + \beta E_{(w', \delta'_k | w)} V_O(w', a', 0, k', \delta'_k, t + 1)\}$$

$$c + g(w, a, 0, k', t) + a' + [1 + \chi_B]p(t)k' = w + a(1 + r).$$

We assume that if an excluded individual purchases a house, he is no longer excluded from the mortgage market (the default flag is removed).¹⁴

¹⁴This assumption is also without much loss of generality because given the substantial transactions costs of purchasing and selling a home, individuals purchase homes and stay in them for a long duration of time. By the time they need to make another purchase, an excluded individual's exclusion flag would typically be gone. Thus, following

If the individual is not excluded from the mortgage market and chooses to rent, he solves

$$M_0(w, a, t) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{u(c, h) + \beta E_{w'|w} V_R(w', a', t + 1)\}$$

$$c + z(t)h + g(w, a, 0, 0, t) + a' = w + a(1 + r),$$

and if he is excluded from the mortgage market and chooses to rent, he solves:

$$M_0^D(w, a, t) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{u(c, h) + \beta E_{w'|w} \lambda V_R(\cdot, t + 1) + (1 - \lambda) V_R^D(\cdot, t + 1)\}$$

$$c + z(t)h + g(w, a, 0, 0, t) + a' = w + a(1 + r),$$

where λ is the probability of reentry into the mortgage market following default. Then $V_R(\cdot, t)$ is given by $\max \{M_1(\cdot, t), M_0(\cdot, t)\}$ and $V_R^D(\cdot, t)$ is given by $\max \{M_1^D(\cdot, t), M_0^D(\cdot, t)\}$. We denote the decision rules of a nonexcluded renter by $c_R(a, w, t)$, $h_R(a, w, t)$ and $k'_R(a, w, t)$, and those of an excluded renter by $c_R^D(a, w, t)$, $h_R^D(a, w, t)$ and $k'_R^D(a, w, t)$. Here, it is understood that $h_R(a, w, t)$ and $k'_R(a, w, t)$ cannot be simultaneously positive (similarly for $h_R^D(a, w, t)$ and $k'_R^D(a, w, t)$).

A homeowner may keep the current house, sell it, or default on the mortgage (if he has one).¹⁵

If he chooses to keep the house, he solves:

$$K_0(w, a, x, k, \delta_k, t) = \max_{c \geq 0, a' \geq 0} \left\{ u(c, k) + \beta E_{(w', \delta'_k)|w} V_O(w', a', x\mu/(1 + \pi'), k, \delta'_k, t + 1) \right\}$$

$$c + g(w, a, x, k, t) + a' + x + \delta_k p(t)k = w + a(1 + r),$$

where we denote the (anticipated) inflation rate between period t and $t + 1$ by π' . We assume that a homeowner must cover the depreciation on the house.

a purchase, excluded individuals will tend to behave as if they do not have a default flag.

¹⁵For computational tractability, we assume that mortgages are issued only at the time of purchase and are terminated only at the time of sale or at the time of default. Thus, a homeowner cannot refinance an existing mortgage or issue a new one against his home if there isn't one currently.

If he chooses to sell, he solves:

$$K_1(w, a, x, k, \delta, t) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{u(c, h) + \beta E_{w'|w} V_R(w', a', t + 1)\}$$

$$c + z(t)h + g(w, a, x, 0, t) + x + a' + \delta_k p(t)k = w + a(1 + r) +$$

$$[1 - \chi_S]p(t)k - q(t)\mu x / (1 + \pi'),$$

where χ_S is the percentage cost of selling a house and π' is the inflation rate between t and $t + 1$. Observe that a seller must pay his current mortgage payment, buy back the promised sequence of future mortgage payments at the nominal risk-free interest rate, and move out (i.e., rent housing space in the period of the sale). The arguments of the tax function reflect these assumptions: The current period mortgage interest payment $x - (1 - \mu)q(t)x / (1 + \pi')$ is deducted from taxes, but since the seller does not consume the services of the house, he does not pay property taxes. A seller must also make good on the depreciation of the house.

If the homeowner has a mortgage, he may choose to default. In this case, he solves:

$$K_D(w, a, x, k, t) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{u(c, h) + \beta E_{w'|w} [(1 - \lambda)V_R^D(\cdot, t + 1) + \lambda V_R(\cdot, t + 1)]\}$$

$$c + g(w, a, 0, 0, t) + a' + z(t)h = w + a(1 + r).$$

Foreclosure results in the individual losing the house as well as the mortgage and in his being excluded from the mortgage market for some random length of time. Importantly, a defaulter does not cover the depreciation cost. Finally,

$$V_O(w, a, x, k, \delta_k, t) = \max \{K_0(w, a, x, k, \delta_k, t), K_1(w, a, x, k, \delta_k, t), K_D(w, a, x, k, t)\}.$$

We denote the decision rules of a homeowner by $c_O(a, w, x, k, \delta_k, t)$, $h_O(a, w, x, k, \delta_k, t)$ and $k'_O(a, w, x, k, \delta, t)$. Again, it is understood that $h_O(a, w, x, k, \delta_k, t)$ and $k'_O(a, w, x, k, \delta_k, t)$ cannot simultaneously be positive. Furthermore, if h_O is positive, the individual is either a seller or a defaulter. We let $d(a, w, x, k, \delta_k, t)$ and $s(a, w, x, k, \delta_k, t)$ be indicator variables signifying default and sale, respectively.

It is worth pointing out that we assume that mortgages are issued only at the time of purchase

and are terminated only at the time of sale or at the time of default. Thus, a homeowner cannot refinance an existing mortgage or issue a new one against his home if there isn't one currently. This is done for computational tractability.¹⁶

3.2 Financial Intermediaries

The (representative) financial intermediary rents out the rental housing stock, accepts deposits, and buys mortgages. The rental housing stock has no other use, so the intermediary simply supplies whatever it owns at the rental price $z(t)$. The intermediary receives $z(t) - \rho p_R(t)$ per unit of housing space in period t , where $p_R(t)$ is the price of a unit of rental housing space. Since the intermediary can always buy or sell rental housing space, $p_R(t)$ satisfies the recursion:

$$p_R(t) = z(t) - (\rho + \Delta)p_R(t) + p_R(t+1)]/(1+r). \quad (6)$$

With regard to deposits, competition leads the financial intermediary to offer the risk-free rate r_f on both taxable and tax-exempt deposits.

With regard to mortgages, competition leads the financial intermediary to charge a price that in expectation earns zero profits. When the intermediary acquires a mortgage, it gives up $q(w, a', x', k'; t) \cdot x'$ in goods. Next period, if the homeowner defaults, the intermediary receives $p(t+1)[1 - \chi_D]k'$, where χ_D is the cost of foreclosure to the intermediary; if the homeowner sells, the intermediary receives $x' + q(t+1)\mu x'/(1 + \pi'')$; and if neither happens, the intermediary receives x' plus the value of the continuing mortgage, which is given by $q(w', a'', \mu x'/(1 + \pi''), k'; t+1)\mu x'/(1 + \pi'')$, where π'' is the inflation rate between periods $t+1$ and $t+2$. The requirement of

¹⁶Note, however, that in our model, as long as the risk-free rate does not fall, the option to refinance the mortgage (meaning prepaying the existing loan and replacing it with a loan of identical size) is not valuable. The reason for this is that the stream of payments on the existing loan is discounted at the risk-free rate when it is prepaid, while there would typically be risk premium on the new loan.

zero profits then reduces to:

$$\begin{aligned}
q(w, a', x', k', t) x' &= (1 + r_f)^{-1} \times & (7) \\
E_{w', \delta' | w} \{ &d(w', a', x', k', \delta', t + 1) p(t + 1) [1 - \chi_D] k' + \\
s(w', a', x', k', \delta', t + 1) &[x' + q(t + 1) \mu x' / (1 + \pi'')] + \\
(1 - d(\cdot, t + 1)) &(1 - s(\cdot, t + 1)) [x' + q(w', a'', \mu x' / (1 + \pi''), k', t + 1) \mu x' / (1 + \pi'')] \}.
\end{aligned}$$

3.3 Developers

Developers, if they own any housing stock in the initial period, choose how much of their stock to sell on the market. We imagine there is a representative developer that acts competitively. Let n denote the stock of unsold homes in the hands of the developer at the start of the current period. Let $F(n, t)$ denote the value function of the developer. Then, the developer solves

$$F(n, t) = \max_{n' \in [0, n]} p(t)(n - n') - (\rho + \Delta)p(t)n' + (1 + r_f)^{-1} F(n', t + 1) \quad (8)$$

The first term in the current return is the revenue from the sale of property, and the second term is the cost — in terms of depreciation and property taxes — of unsold properties. We denote the decision rule of the developer by $n'(n, t)$.

4 Equilibrium

An equilibrium consists of a stock of rental housing H_R , a stock of owner-occupied housing H_O , initial distributions of excluded and nonexcluded renters over individual states $\mu_R(w, a, 0)$ and $\mu_R^D(w, a, 0)$, an initial distribution of homeowners $\mu_O(w, a, x, k, \delta_k, 0)$, the initial holdings of developers $n(0)$, a sequence of strictly positive rents $\{z^*(t)\}$, a sequence of rental housing prices $\{p_R^*(t)\}$, a sequence of owner-occupied housing prices $\{p^*(t)\}$, deposit interest rate r_f and \bar{r} , a sequence of mortgage price functions $\{q^*(w, a', x', k', t)\}$, a sequence of decision rules, a sequence of distributions $\mu_R^*(w, a, t)$, $\mu_R^{*D}(w, a, t)$ and $\mu_O^*(w, a, x, k, \delta_k, t)$, $t \geq 1$, and a sequence of inventory holdings of developers $\{n_t^*\}$, $t \geq 1$, such that:

1. The decision rules are optimal, given $r_f, \bar{r}, z^*(t), p^*(t), q^*(t)$.
2. $\{p_R^*(t)\}$ satisfies (6) and $\{q^*(w, a', x', k', t)\}$ satisfies (7).
3. Demand for rental housing equals supply for all $t \geq 0$

$$\int h_R^*(w, a, t) \mu_R^*(dw, da, t) + \int h_R^{D*}(w, a, t) \mu_R^{*D}(dw, da, t) + \int h_O^*(a, w, x, k, \delta, t) \mu_O^*(da, dw, dx, dk, d\delta_k, t) = H_R.$$

4. Demand for owner-occupied housing equals supply for all $t \geq 0$

$$\int k_R^*(w, a, t) \mu_R^*(dw, da, t) + \int k_R^{D*}(w, a, t) \mu_R^{*D}(dw, da, t) + \int k_O^*(a, w, x, k, \delta_k, t) \mu_O^*(da, dw, dx, dk, d\delta_k, t) + n^*(t+1) - n^*(t) = H_O.$$

5. The sequence of distributions $\{\mu_R^*(w, a, t)\}$, $\{\mu_R^{*D}(w, a, t)\}$, and $\{\mu_O^*(w, a, x, k, \delta_k, t)\}$, $t \geq 1$, are implied by the sequence of optimal decision rules and initial distributions $\mu_R(w, a, 0)$, $\mu_R^D(w, a, 0)$ and $\mu_O(w, a, x, k, \delta_k, 0)$ and the sequence of inventory holdings of developers $\{n^*(t)\}$, $t \geq 1$, is implied by the optimal decision rule of developers and their initial holdings $n(0)$.

5 Parameter Selection and Calibration

Turning first to the Markov process for earnings, we assume that log earnings follow an AR1 process:

$$\ln(w_{t+1}) = \bar{w} + \psi \ln(w_t) + \epsilon_{t+1} \tag{9}$$

Several studies have estimated log earnings processes for the U.S. using PSID earnings data.¹⁷ Estimates of ψ and the standard deviation of ϵ (σ_ϵ) vary across studies. We follow Storesletten,

¹⁷These processes are typically modeled as the sum of a fixed random effect, an AR1 process, and a purely transitory shock. For reasons of tractability, we ignore the fixed random effect and the purely transitory shock.

Telmer, and Yaron (2004a) and Storesletten, Telmer, and Yaron (2004b) in setting $\sigma_\epsilon = 0.129$ and $\psi = 0.97$.

Setting aside the parameters of the income tax schedule, our model economy has 15 other parameters. These include three preference parameters (β, θ, γ) , six parameters related to housing transactions $(\chi_S, \chi_B, \psi, \Delta, \delta_k, \xi)$, one related to the mortgage contract (μ) , two related to the costs of foreclosures (λ, χ_D) , and three related to the asset market (ω, r_f, \bar{r}) , and, finally, the steady-state inflation rate (π) .

Of the preference parameters, γ is set to 2, which is a standard value in macro studies, and the value of θ is set to 0.15 based on the NIPA share of nominal housing expenditures in nominal personal consumption expenditures.¹⁸

Of the foreclosure-related parameters, the value of λ was set to 0.25, which implies an average exclusion period following default of four years.¹⁹ The loss in the value of a house that goes into default is set to 17 percent, which fixes χ_D to 0.17.²⁰

Of the housing transactions parameters, Gruber and Martin (2003, p. 19) find (from the Survey of Consumer Expenditures) that the median household reported selling costs of 7.5 percent and buying costs of 2.5 percent of the house value. We assume that the total cost of selling a house is 7 percent of the house value and split this into a 6 percent selling cost and a 1 percent buying cost, which fixes χ_S and χ_B , respectively.²¹ The average property tax rate in the U.S. in 2007 was 1.38 percent, so ψ was set to 0.0138.²² The random depreciation shock for homeowners is set to χ_D , so $\delta = 0.17$.²³

¹⁸This share has been roughly stable over the 1929-2012 period. The average for the 1929-2006 period is 0.1455, and the average for 1990-2006 is 0.1510.

¹⁹We chose a relatively short exclusion period because lenders may well lend to a household with a foreclosure in its credit history as long as the household is willing to put down enough down payment on the mortgage.

²⁰Shilling, Benjamin, and Sirmans (1990, Exhibit 1, p. 136) document that the price per square foot of foreclosed properties is about 11 percent less than that of nondistressed properties. Employing a larger data set, Pennington-Cross (2006, p. 211) reports that distressed properties appreciate about 22 percent less than other properties do. We set χ_D to the midpoint of these two values.

²¹Since transaction costs of buying and selling a house may be shared between buyers and sellers, it may not be appropriate to simply sum up the percentage costs reported by buyers and sellers separately. For this reason, we use a somewhat lower total transaction cost of 7 percent.

²²As reported in www.nytimes.com/2007/04/10/business/11leonhardt-avgproptaxrates.html

²³The depreciation shock leads to a default when the home-equity ratio is low enough. Thus, we assume the same loss in home value as in a foreclosure.

Based on recent trends, the inflation rate π was set to 0.025.²⁴

Turning to the asset market parameters, we set the real pretax return on financial assets to 4 percent, which fixes r_f to 0.04. Regarding the tax treatment of interest earnings, we recognize that only a portion of the nominal returns on financial assets is taxed at the relevant individual income tax rate; the remaining portion is taxed at a (potentially) lower rate because some of the return on assets is in the form of capital gains (which are typically taxed at a lower rate). In addition, for assets that are in retirement accounts, capital gains and dividends as well as interest payments are not taxed until the individual reaches retirement. We assume that the portion that is taxed at the relevant income tax rate is 40 percent, which sets ω to 0.40.²⁵ We assume that the returns on the remaining portion are taxed at a flat rate of 20 percent after a period of 10 years. Given an inflation rate of 2.5 percent and a real return of 4 percent, this is equivalent to annual aftertax earnings from long-term investments of 2.973 percent, which fixes $r_e = 0.0297$.²⁶ These values are summarized in Table 1.

We need to specify the tax schedule $\tau(\cdot)$ and the standard deduction s . The tax schedule is chosen to match the tax table for 2001. In our model, people are viewed as individuals, which is consistent with the earnings data, but we will view individuals as being married. Hence, the tax table we use is the tax table for married, filing separately. According to the Census Bureau, the median income of year-round full-time workers age 25 and older in 2001 was \$30,969. Normalizing the tax brackets for 2001 by this estimate of median income, we obtain tax schedule $\tau(\cdot)$ given in Table 2. And, normalizing the 2001 standard deduction for a married person filing separately by median income gives $s = 0.123$.²⁷

²⁴The CPI inflation rate has tended to drift downward in the post-WWII era. Between 1947 and 2005, it averaged 3.93 percent; between 1987 and 2005, it averaged 3.14 percent; and between 1997 and 2005, it averaged 2.51 percent.

²⁵Between 2001 and 2009, the fraction of household financial assets in retirement accounts was roughly stable around 35 percent (Investment Company Institute (2009, p. 5)). Of the remaining 65 percent, we assume that 70 percent is allocated to equity. The return on equity due to capital gains has been about 58 percent (Ibbotson and Chen (2003, Figure 1)). Thus, the portion of return on financial assets that is taxed at a lower rate is $0.35 + (0.65)(0.70)(0.58) \approx 0.60$.

²⁶The nominal gross aftertax return on a dollar invested in the long-term asset is $[(1.025 \times 1.04)^{10} - 1](1 - 0.20) + 1 = 1.6890$, and the real return is $1.6890/(1.025)^{10} = 1.3194$, which implies an aftertax real rate of return of $1.3194^{1/10} = 1.02973$. Hence, $r_e = 0.0297$.

²⁷Our tax schedule overstates the taxes paid by low-income people because we ignore the earned income tax credit (EITC). However, what is important for our study is the tax benefit of owner-occupied housing, and this benefit is *not* affected by the EITC. This is because the credit is calculated on a person's adjusted gross income and, therefore, does not depend on whether the household rents or owns.

Table 1: Parameters Selected Independently

Parameter	Value	Description
λ	0.25	Probability of reentry after default
ψ	0.97	Autocorrelation of earnings
σ	0.129	Sd of innovation to earnings shock
δ	0.17	Depreciation shock for homeowners
θ	0.15	Exponent to housing consumption
γ	2.0	Risk-aversion coefficient
π	0.025	Steady-state inflation
χ_B	0.01	Cost of buying
χ_S	0.06	Cost of selling
χ_D	0.17	Foreclosure cost
ρ	0.0138	Property tax rate
r_f	0.04	Risk-free real interest rate
r_e	0.02973	Real aftertax annual return on long-term investment
ω	0.40	Portion of asset return that is currently taxable

Table 2: Tax Function

Tax Brackets	Tax Rate
0.00 - 0.73	0.15
0.73 - 1.76	0.28
1.76 - 2.68	0.31
2.68 - 4.80	0.36
4.80 -	0.39

The remaining four parameters $(\beta, \Delta, \xi, \mu)$ are determined by matching model moments with selected data moments. The selected moments are the homeownership rate in 2007 as reported by the Census Bureau,²⁸ the fraction of homeowners with home equity less than or equal to 25 percent as reported in the 2007 Survey of Consumer Finances excluding the top 3 percent most wealthy households (as measured by net worth),²⁹ the average foreclosure rate between 1991:Q1-2006:Q1 as reported by the Mortgage Bankers Association, and the fraction of homebuyers who bought with cash at the start of 2006 as estimated by Goldman Sachs.³⁰

The model moments are computed for the steady state of the model. To solve for the steady state, we normalize z^* to 0.25 (any other value would do just as well) and $p_R^* = z^*/(1 - q + \rho + \Delta)$, where $q = 1/(1 + r_f)$. It is assumed that owner-occupied housing sells for the same price as rental housing in the steady state.³¹ Given these prices, the implied demand for rental and owner-occupied housing determines the aggregate stocks H_R and H_O , respectively.

The results of the matching exercise are displayed in Table 3. The model matches the target statistics exactly. The parameter values that achieve this match are listed in the final column. Although these statistics are jointly targeted, the parameter listed in each row is the one that is most determinative for the corresponding statistic.

Table 3: Parameters Selected Jointly

Targeted Statistics	Data	Model	Parameter Values
Homeownership rate	0.68	0.68	Rental depr. rate, Δ 0.0167
Steady-state foreclosure rate	0.015	0.015	Prob depr. shock, ξ 0.064
Frac of homeowners with $\leq 25\%$ equity	0.18	0.18	Mortgage decay, μ 0.988
Frac of cash buyers	0.19	0.19	Discount factor, β 0.947

Of the four parameters selected by this procedure, Δ , ξ , and μ have real-world counterparts. Shilling, Sirmans, and Dombrow (1991) estimate the depreciation rate for owner-occupied and rental

²⁸We chose the 2007 homeownership rate as the target in order to be consistent with the timing of the Survey of Consumer Finances closest to the start of the crisis.

²⁹As is well known, this class of models cannot easily match the upper tail of the wealth distribution. Since mortgage default is not a phenomenon that afflicts the wealthy, we abstract from the upper tail of the wealth distribution in this paper. The home-equity ratio is defined as the value of the home minus housing debt to the value of the home.

³⁰Goldman Sachs, *The Mortgage Analyst*, Credit Strategy Research, August 14, 2013, Exhibit 4, p. 3

³¹By equalizing the steady-state prices of both types of housing stocks, we are assuming that long-run construction costs of housing are constant and, in the long run, the stock of both types of housing adjusts to equal the amounts demanded.

properties. They find that the average depreciation rate for owner-occupied properties declines with age and is 1.06 percent per year in the 10th year of use, while for rental properties, it is 1.66 percent per year in the 10th year of use. The value of Δ almost exactly matches the depreciation rate of rental properties that are 10 years old. For owner-occupied housing, the values of ξ and δ imply an average annual depreciation rate of 1.08 percent, which is also very close to the estimated depreciation rate of 10-year-old owner-occupied dwellings.

The mortgage decay parameter implies an average duration of a mortgage of 80 years, much longer than the duration of any mortgage issued in the U.S. If we set μ to get a mortgage with an average duration of, say, 22 years (average of 15 and 30 years) and leave all other parameters unchanged, households accumulate home equity much faster in the model than they do in the data. Presumably, this is because many households periodically extract equity from their homes via refinancing, second mortgages, and home equity lines of credit (HELOCs). Since these extraction margins are not present in our model, matching the home equity distribution requires a counterfactually long mortgage duration.

6 Analysis of Steady State

Table 4 lists some statistics that were not targeted but are relevant for judging the validity of the model. The data on the ratio of the median square-footage-per-person of owner-occupied dwellings to median square-footage-per-person of rented dwellings are obtained from the 2007 American Community Survey. All other statistics are obtained from the 2007 SCF (top 3 percent most wealthy excluded). Overall, the model's performance seems reasonable. One dimension in which it does poorly is the financial wealth-to-income ratio, with people accumulating less financial wealth in the model.³² However, given that our model features only one source of uncertainty (earnings), the precautionary savings motive is weaker in the model than in reality.

We now analyze some the key forces at work in our model to shed light on the similarities and differences between the model and the data displayed in Table 4.

³²Financial wealth is defined as financial assets - credit card balance - margin loans, loans against pensions, loans against life insurance - other lines of credit not secured by equity in home - educational installment loans.

Table 4: Model Performance

Nontargeted Statistics	Data	Model
Avg. inc. of homeowners/avg. income of renters	2.15	1.83
Avg. housing wealth/avg. income	1.69	1.14
Avg. financial wealth/avg. income	1.83	0.71
Ratio of median owner-occupied to rental sq. ft/person	1.51	2.07
Average home equity ratio	0.62	0.67
% of homeowners with $\leq 0\%$ equity	1.81	0.51
% of homeowners with $\leq 10\%$ equity	7.02	7.03
% of homeowners with $\leq 20\%$ equity	14.07	14.47
% of homeowners with $\leq 30\%$ equity	22.40	20.34
% of homeowners with 100% equity	28.75	34.21

6.1 Taxes and Homeownership

In our model, owner-occupancy does not provide any utility benefit per se, and there are significant transaction costs for purchasing and selling homes (with the attendant lack of flexibility for adjusting housing consumption to earnings shocks) as well as the risk of the depreciation shock. Nevertheless, more than two-thirds of individuals purchase their homes. There are two reasons for this. First, owner-occupancy is more efficient in delivering housing services than renting: The average depreciation rate of owner-occupied dwellings is lower than the average depreciation rate for rentals. Second, owner-occupancy has tax advantages. The implicit rental income from ownership is not counted as part of income and therefore not taxed; this exemption means that people — especially those in the higher tax brackets — have a strong incentive to own homes.³³ The deductibility of mortgage interest payments encourages individuals to *borrow* to finance the purchase of their homes (as opposed to paying for the purchase from accumulated assets). Both tax effects operate more strongly for higher-income individuals.

Since the tax effects work more strongly for higher-income individuals, our model predicts that homeownership should be concentrated among higher-income households. Indeed, in our model, the average income of homeowners is 1.83 times the average income of renters, which compares

³³Let's say the household is deciding between saving in a risk-free asset or saving in a home. When the household saves in a risk-free asset, it pays taxes on the nominal interest return. If the household saves by buying a house, the return to that saving comes as (implicit) rental income and appreciation in the value of the house, both of which are not taxed. So, there is a tax benefit to homeownership.

favorably with the data, where it is 2.12. Our model also predicts that homeowners consume more housing space, on average, than renters. High earners choose to buy houses, which makes the housing space of owner-occupants larger than that of renters; in addition, the tax benefits of owner-occupancy makes owner-occupants consume more housing than renters (more on this point later). The mean per-capita housing space of owner-occupants is 2.07 times that of renters, higher than it is in the data. This is to be expected, as the income of homeowners is almost twice that of renters and, in addition, there are tax incentives for homeowners to consume housing space. In reality, the additional housing expenditure of homeowners is spent on higher-quality housing in addition to more space.

If owner homeownership is tax advantaged, why don't individuals save only in the form of houses? The reason is that the higher implicit return on housing must be balanced against the fact that the higher return must be spent on housing consumption. Thus, a homeowner's investment in home equity is bounded by the utility flow from housing services. In our calibration, the exponent to housing services in the Cobb-Douglas utility function is 0.15, which implies sharply diminishing marginal utility from housing services and results in average housing wealth to average income of 1.14, somewhat lower than what we find in the data.

Table 5: Itemization and LTV Pattern

Income/Median Income	≤ 0.92	$0.92 - 1.72$	$1.72 - 2.87$	$2.87 - 5.74$	≥ 5.74
	Data				
% that itemizes	23.4	66.1	85.5	98.4	99.9
Loan-to-value ratio in %	25.9	44.9	47.4	42.6	29.4
	Model				
% that itemizes	12.4	33.0	35.3	36.0	n/a
Loan-to-value ratio in %	33.6	35.0	27.9	27.5	n/a

Our model also predicts that the fraction of individuals who itemize their federal taxes is increasing in income as is the LTV ratio of homeowners who buy with a mortgage. Table 5 compares these predictions with those reported by Poterba and Sinai (2008, Table 1, p. 85). The itemization rate is lower in the model than in the data, consistent with the fact that owner-occupancy is but one reason for itemization. Importantly, the model itemization rate is increasing in income, which matches the pattern in the data. The model overpredicts the LTV ratio of low earners relative to

the data.

Table 6: Effects of Tax Treatment of Housing

Statistic	Baseline	No Mtg. Ded.	Taxes on Implicit Rents
Avg. housing consumption	1.0	0.94	0.88
Avg. home equity ratio	0.67	0.78	0.47
Homeownership rate	0.68	0.73	0.56

Table 6 shows how the steady state is altered if these tax benefits are reduced. If the mortgage deduction is eliminated, the average housing space consumed declines by 6 percent; if the implicit rental income is taxed (but the homeowner is allowed to deduct mortgage interest payments and property taxes), the average space consumed declines by 12 percent.

Table 6 also shows interesting effects of the tax treatment of housing on home equity and the homeownership rate. Eliminating the mortgage deduction blunts the incentive to borrow to purchase a home, and home equity ratio rises substantially. Taxing implicit rental income (but allowing deductions for mortgage interest and property taxes) increases leverage and reduces average home equity. If the mortgage deduction is eliminated, the homeownership rate *rises*, which is surprising. This happens because less leverage leads to fewer foreclosures and, hence, higher owner-occupancy. If implicit rental income is taxed, the homeownership declines to 56 percent.

Table 7: Effects of Inflation

Statistic	$\pi = 0.01$	Baseline	$\pi = 0.04$
Avg. housing consumption	0.97	1.00	1.03
Avg. home equity ratio	0.62	0.67	0.65
Homeownership rate	0.79	0.68	0.65

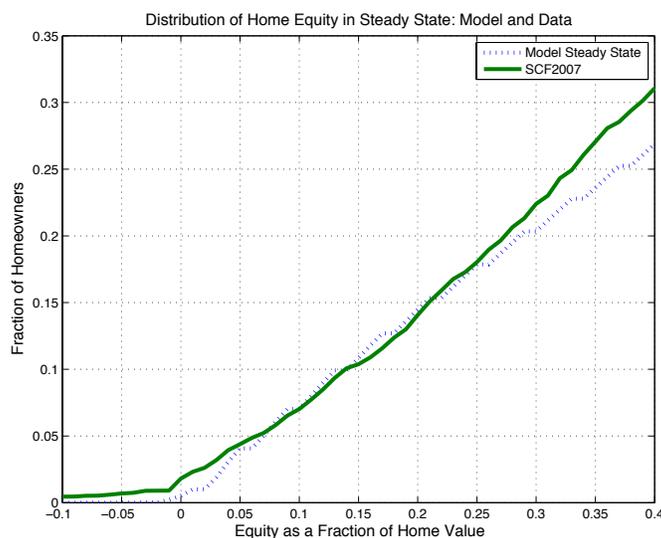
Because taxes are computed on nominal income, the inflation rate affects the tax benefits of owner occupancy. Table 7 reports the effects of alternative steady-state inflation rates. A higher inflation rate has two direct effects on the housing market. First, it increases the nominal interest rate and, therefore, increases the tax benefits of the mortgage deduction, since it is nominal (not real) interest payments that are tax deductible. This encourages homeowners to leverage up and buy bigger houses. Second, higher inflation erodes the value of debt faster and thus causes households to accumulate home equity at a faster rate. In our model, the first effect dominates and average

home equity decreases. The homeownership rate also goes down because more leverage means more defaults.³⁴ These effects tend to work in the reverse when steady-state inflation drops. The exception is average home equity, which drops with lower inflation as well.

6.2 What Factors Determine the Home-Equity Distribution in the Model?

For the distribution of home equity across all homeowners, geometric decay and inflation play important roles. Because the mortgage contract is nominal, inflation steadily reduces the real value of debt over time and, therefore, steadily increases the real value of home equity. In addition, households steadily pay down their debt at a constant geometric rate. The almost linear shape of the CDF of the home equity distribution — both in the model and in the data — result from these two steady forces at work. Figure 1 displays the steady-state home-equity distribution in the model and the data.

Figure 1: Lower Tail of the Home Equity Distribution

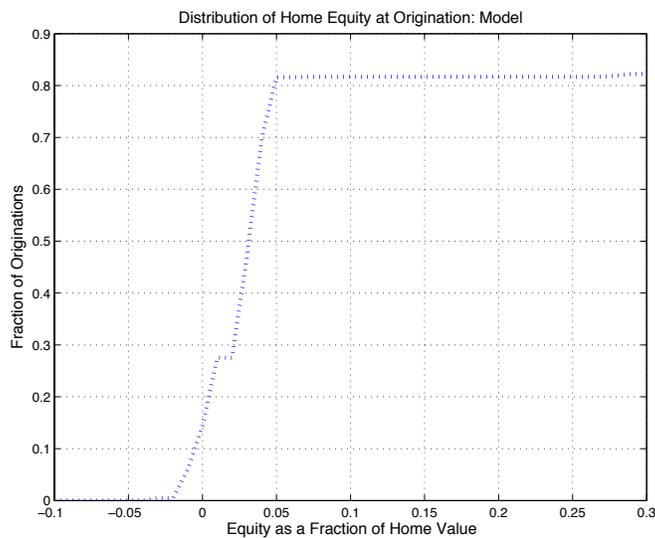


The home-equity distribution at origination (i.e., among buyers who take out a mortgage) is concentrated between -5 percent and $+5$ percent, as shown in Figure 2. Individuals who borrow to purchase homes typically itemize and, therefore, find it profitable to leverage up to the point

³⁴Higher inflation also makes saving in housing more attractive as the effective tax rate on financial assets becomes higher when inflation is higher, since those taxes also depend on the nominal return.

where the interest rate on the mortgage begins to reflect a measurable default-risk premium.³⁵ While counterfactual, high leverage at the time of purchase seems to be a necessary ingredient to matching the lower tail of the steady-state home-equity distribution, given that we do not allow homeowners to extract equity from their homes without selling them (no refinance option).

Figure 2: CDF of Home Equity at Origination



6.3 Why Is There Default in the Steady State?

For there to be default on a mortgage, it is necessary for the selling option to be inferior to default — which can happen only if the value of the homeowner’s obligation to the lender exceeds the sale price of the house less the transaction costs of selling. Thus, negative *net* home equity is a necessary condition for default. But it is not sufficient because the default option has to dominate both the option to sell and the option to keep the house.

For default to dominate selling, the costs of default must be less than the capital loss imposed by selling. The costs of default stem from the loss of access to mortgage markets (for some length

³⁵Publicly available data on home equity at origination at time of purchase, including any second mortgages or HELOCs taken out at the same or almost the same time, is not available. Based on proprietary data in Elul, Souleles, Chomsisenghet, Glennon, and Hunt (2010), the fraction of all prime fixed-rate purchase mortgages issued in 2006 with home equity ratio between -5 and $+5$ percent (taking into account second mortgages and HELOCs obtained within six months of the origination of the first mortgage) was around 22 percent (private communication); in the model, this statistic is around 82 percent. However, prime fixed-rate mortgages accounted for about 64 percent of all purchase mortgages issued in 2006, and other mortgage products (nonprime fixed-rate mortgages, adjustable rate mortgages, etc.) tend to have higher loan-to-value ratios (lower home equity ratios) than fixed-rate mortgages.

of time). If the defaulter does not have the personal wealth to purchase a house, this loss of access implies the loss of the tax benefits of homeownership. If the capital loss from selling is small relative to the tax benefits of homeownership, the default option may not dominate the selling option.

In addition, even if default dominates selling, the option of keeping the house may dominate default. For default to dominate the keeping option, utility from keeping must be relatively low. This will tend to be the case if, since taking out the mortgage, the individual's income has changed sufficiently to make the size of his house and/or the size of his mortgage suboptimal relative to his current resources. In particular, if he has experienced bad income shocks, his house and/or mortgage may be too large relative his earnings, and, thus, the keeping option may be low relative to the default option. Also, since the cost of default tends to be lower for low earners, the default option is relatively attractive for such a borrower.

In the steady state with a constant price of housing, there are essentially two ways in which an individual can end up with negative net home equity. One way is for the individual to knowingly borrow more than the value of the house, less selling costs, and many do (recall that home equity at origination is concentrated between -2 percent and 5 percent). Some in this group will default if they are hit with a series of negative income shocks soon after taking out the mortgage. But such events are rare and initial leverage contributes very little to the foreclosure rate.

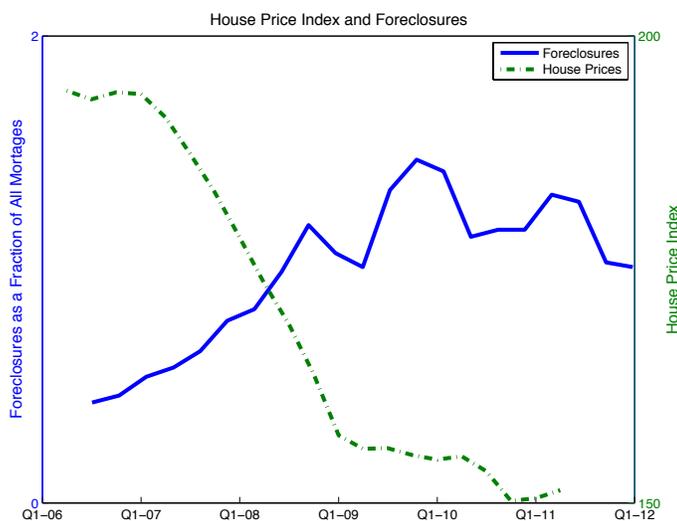
The second way for negative net home equity to occur is if there is an *idiosyncratic* loss in the value of the house offered as collateral. The random depreciation shock, δ , allows for this possibility. When the depreciation shock hits, a homeowner with relatively low home equity ends up with substantially negative (net) home equity. But since a borrower can control home equity via his down payment, depreciation shocks per se do not necessarily generate foreclosures. It must also be the case that foreclosure of a depreciated property does not lead to additional depreciation; i.e., $\chi_D = \delta$. Then, it is efficient for risk-neutral lenders to insure risk-averse borrowers against the occurrence of this i.i.d. shock. If the shock happens, lenders take the property back through a foreclosure and, in return, charge a higher interest payment spread through the years.³⁶

³⁶If foreclosure imposes additional costs on lenders, i.e., $\chi_D > \delta$, lenders will ask for a higher premium and the cost of insurance will rise beyond what is actuarially fair from the perspective of the homeowner. This will motivate homeowners to provide a larger down payment and thereby lower (perhaps eliminate) the probability of default on the loan.

7 Accounting for the Foreclosure Crisis

In this section, we use the model to account for the foreclosure crisis. The key features of this crisis are displayed in Figure 3; namely, the fall in house prices and the rise in foreclosures since early 2006. House prices dipped around 2006-Q2 and then, except for a small rise in early 2007, fell continuously until 2009-Q2. At that point, house prices stabilized for about year, fell again for half a year, and eventually began to rise. The rate of new foreclosures rose continuously between 2006-Q2 and 2008-Q4 and has fluctuated around the high value reached at that time for the better part of three years before showing a tendency to decline. We summarize this history as a 19 percent drop in house prices and 16 percent foreclosure rate over the course of the crisis.³⁷

Figure 3: House Prices and Foreclosure Rates 1991-2011



As noted in the Introduction, we consider three shocks in our accounting, all of which occur in period 1 and are unanticipated as of period 0. The shocks include (i) an increase in the stock of housing meant for owner-occupancy, (ii) an increase in the duration of the foreclosure process that allows defaulters to stay rent-free in their to-be-foreclosed home, and (iii) an increase in financial

³⁷The drop in house prices is computed from the CoreLogic house price index excluding distressed sales. According to this series, house prices peaked in 2006Q2 and then fell and temporarily stabilized in 2009 Q2. We use three-quarter averages centered around these peak and trough quarters to calculate the percentage decline in price. For the foreclosure rate, if we sum the quarterly new foreclosure starts rate between 2006 Q2 to 2010 Q2, we obtain foreclosures totaling 16 percent (we cumulate up to 2010 Q2 because foreclosures take time to process, and the foreclosures that occurred in 2010 Q2 presumably started around 2009 Q2 or earlier). If we include another year's worth of foreclosures, we obtain 19 percent.

friction, which implies an increase in interest rates on new mortgages.

We model the increase in the housing stock as a positive holding of housing space in hands of developers in period 1, i.e., $n(1) > 0$. McNulty (2009) reports that between 2005 and 2007, the housing stock increased by 3.8 million units, but the number of occupied housing units increased by only 1.8 million units. Thus, about 2 million housing units were added that did not have occupants. Since houses typically sit on the market for some time before they are occupied, part of the increase in unoccupied housing units is simply a reflection of “frictional” vacancy. McNulty estimates the increase in unoccupied units because of frictional vacancies to be about 0.28 million units, which leaves an excess of 1.72 million units. As a percentage of the stock of owner-occupied housing units in 2005, this is about 2.3 percent. We set $n(1) = 0.03 \times H_O^*$. We chose a somewhat higher excess supply to compensate for the fact that our model leaves out features that, in the real world, tend to lower the elasticity of housing demand with respect to the price of housing space.³⁸

We model the financial friction as a “tax” on borrowers, $\Psi(t)$, such that if a household makes a promise to pay the sequence $\{x', \mu x' / (1 + \pi''), \dots\}$, it obtains $q(w, a', x', k', t)(1 - \Psi(t))x'$ in the current period. We assume that this wedge remains constant for periods 1–4 and then declines at the rate of 20 percent per period. We chose the size of the initial wedge so that the model produces a decline in the price of owner-occupied housing of 19 percent in period 1. Calibrated in this way, $\Psi(1) = 0.132$, which is roughly equal to a 1-percentage-point increase in the cost of funds beyond the risk-free rate. Hall (2011, Table 2) reports that the spread between AAA corporate bonds and constant-maturity 20-year Treasuries rose 1.08 percentage points during the worst of the crisis. He interprets the widening spread between two essentially default-free debt instruments as reflecting the emergence of a financial friction “wedge.” Our calibration of Ψ is consistent with this evidence.

We model the lengthening of the foreclosure process as a positive probability of not having to move out of the house in the period of default. In normal times, a foreclosure takes about 6 months to complete, but during the crisis, foreclosures have been taking an additional 7.5 months on average.³⁹ Based on this, the probability of not having to move out in the period of default is

³⁸For instance, in our model, a drop in the price of housing space leads to an increase in the measure of small owner-occupied houses. In reality, the measure of different-sized houses is unlikely to change much when house prices drop.

³⁹We compared the average days delinquent for foreclosure in August 2010 (468 days) with January 2008 (249) days, which implies a lengthening of around 7 months. The data are from Loan Processing Services (LPS); see

set to 0.63 for periods 1–4.⁴⁰

7.1 Baseline Results

Table 8 displays the equilibrium outcome regarding house prices and foreclosures for the new steady state and for the period of the shock (the initial period). In the new steady state, the increase in the supply of owner-occupied housing has benign effects: The 3 percent increase in the supply of owner-occupied housing space leads to a roughly 2 percent decline in the price of owner-occupied housing (and a 3 percent decline in rents). The additional owner-occupied housing stock is absorbed through an increase in the average housing space occupied by owners and a small increase in the fraction of homeowners. There is no measurable change in the steady state foreclosure rate.

Table 8: Crisis, Impact, and Steady State

	SS	Post-shock SS	Impact
House prices	1	0.98	0.81
Foreclosures (%)	1.54	1.54	13.71

In the period of the shock, however, the foreclosure rate jumps to 13.71 percent, about 2.25 percentage points shy of the actual rate of foreclosures during the three-year crisis period. Figure 4 displays the full transition path for prices and foreclosures, along with the paths in the data. In the model, the drop in house prices and the jump in foreclosures happens in the period of the shock. In the periods that follow, the foreclosure rates drop, and house prices rise toward their new steady-state values. In the data, the drop in house prices and the rise in foreclosures happen over several years. Thus, the model is successful in matching the cumulative decline in prices and the cumulative increase in foreclosure over the crisis period but not their dynamic paths.⁴¹

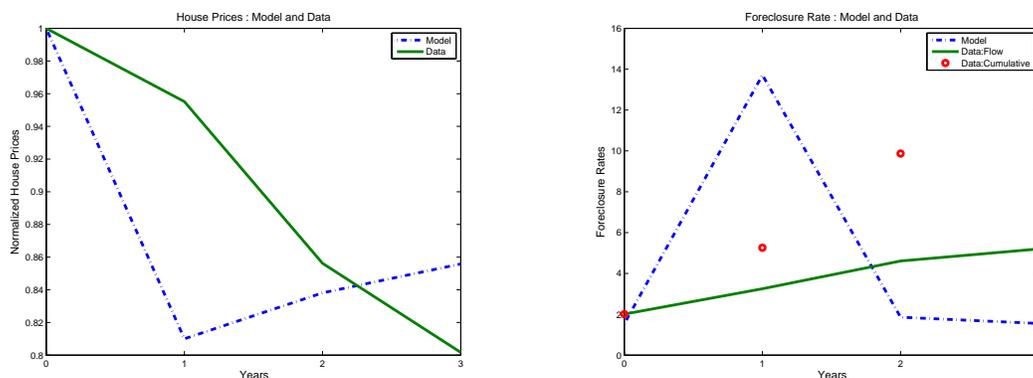
The key forces shaping the model’s transition path are as follows. Developers that own the addition to the housing stock meant for owner occupancy have a strong incentive to sell their

www.lpsvcs.com/LPSCorporateInformation/ResourceCenter/PressResources/Pages/MortgageMonitorArchive.aspx.

⁴⁰While we model the lengthening of the time to foreclosure as an exogenous event, it is possible that the lengthening is a self-fulfilling outcome, wherein a large number of individual borrowers expect processing delays from a high volume of defaults and then default and thus confirm these expectations. See Arellano and Kocherlakota (2014) for a model of sovereign default with this feature.

⁴¹To match the latter will require deviating from the assumption that all shocks occur in period 1 and their equilibrium effects are instantly and correctly perceived by all agents.

Figure 4: Transition Paths, Model, and Data



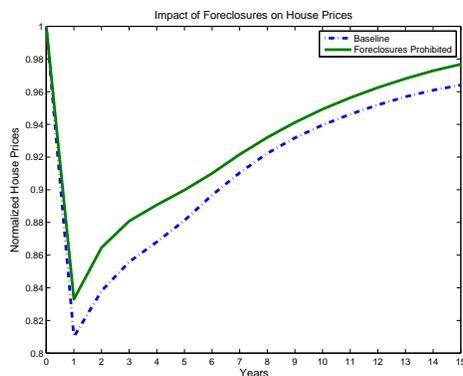
inventory in period 1.⁴² But without any change in house prices or rents, there is no change in the demand for owner-occupied housing. Thus, period 1 house prices fall to induce erstwhile renters to purchase homes.⁴³ However, the absorption of new housing space is hampered by the transaction costs of purchasing (and selling) homes and by the increase in financing costs because of the financial wedge. These frictions force a large drop in the price of owner-occupied house space in period 1. The drop pushes a large fraction of homeowners into negative (net) home equity and some of these homeowners default. The delays in processing foreclosures encourage additional defaults.

In the period following the shock, the price of housing begins its rise back to steady state. Housing is cheap (relative to steady state), and homeowners gradually sell their existing homes and purchase larger ones. The transition is prolonged; it takes 16 years for house prices to get within 1 percentage point of their new steady-state value. Transaction costs as well as the financial wedge (which stays high for four periods and then declines) negatively affect the transition speed. The foreclosure rate stays somewhat elevated for as long as processing delays allow defaulters to live rent free for one year with some probability. Once these delays end, the foreclosure rate drops to essentially its new steady-state level.

⁴²Holding on to empty houses is costly (developers have to cover depreciation and property taxes).

⁴³The drop in price could also motivate homeowners to sell and buy bigger homes, but this channel is not active in period 1.

Figure 5: Transition Paths, with and Without Foreclosures



The rise in foreclosures releases more housing space on the owner-occupancy market, exacerbating the drop in price and inducing further default.⁴⁴ We can get a quantitative sense of the feedback from foreclosures to house prices by computing the transition path with foreclosures permanently prohibited in period 1 onward. This comparison is shown in Figure 5. House prices now fall 16.7 percent on impact. Thus, the model assigns 2.3 percentage points of the 19-percentage-point decline in prices to foreclosures themselves.

7.2 The Contributions of Shocks

In the rest of this section, we quantify the role of different factors to the decline in the price of housing and to the rise in foreclosures. The results are summarized in Table 9.

⁴⁴Whether foreclosures add to the supply of housing space for sale depends on the defaulter's next-best alternative. If the defaulter is choosing between selling and defaulting, preventing him from defaulting would push him to sell. This would imply that preventing foreclosures will *increase* the supply of housing space. On the other hand, if the next-best alternative to default is to keep the house, preventing default will *decrease* the supply of housing space offered for sale. Because of the large drop in price, the next-best alternative to default for most individuals is to keep the house, and foreclosures are a depressive force on house prices.

Table 9: Marginal Contributions of Shocks

Experiment	House Price Decline	Foreclosure Rate
Baseline	19.00	13.71
No financial disruption	5.93	10.51
No processing delays	19.84	8.03
No supply shock	12.33	7.38
Only supply shock	5.46	4.84

7.2.1 The Role of Mortgage Market Disruptions

The foreclosure crisis disrupted the flow of funds into the mortgage market. In the model, this is accounted for by the wedge $\Psi(t)$, which is constant for four years (model periods) and then declines rapidly. If the wedge is eliminated, the price of owner-occupied housing declines by 5.93 percent. The reason is that the renters are more willing to jump in and buy houses when the cost of mortgages is lower and, when they do buy houses, they buy larger ones. Also, because there are fewer foreclosures, there is less downward pressure on the price of owner-occupied housing (this point is explained in the next section). On all these counts, the decline in house prices is much more moderate.

Although the house price decline is moderate, the foreclosure rate remains high: 10.51 percent as opposed to 13.71 percent. One reason default remains attractive is because of the possibility of living rent free following default. A second reason is that defaulters have the option to take out a mortgage and purchase a home once their exclusion period is over, and this option is more valuable when there is no financial disruption (mortgages are offered at the same terms pre- and post-crisis).

Overall, the financial market disruption appears to be the key factor driving the drop in house prices in our model because it contributes (on the margin) nearly 69 percent to the decline in house prices but only 20 percent to the jump in foreclosures.⁴⁵

⁴⁵These (and other reported marginal contributions that follow) are computed relative to the observed decline in house prices and the observed increase in foreclosures. For house prices, it is $(19 - 5.93)/19 \times 100$ and for foreclosures it is $(13.71 - 10.51)/16 \times 100$.

7.2.2 The Role of Lengthened Time to Foreclosure

The fact that the foreclosure process has lengthened considerably during the crisis may have contributed to the crisis itself. We can examine what equilibrium default and price decline would be like if the probability of staying rent free for one year is set to zero. The fraction of mortgages that default in the period of the shock is then only 8.03 percent. Although foreclosures drop, the amount of owner-occupied housing space offered for sale actually *increases* because all foreclosed properties are now offered for sale. The drop in the price of housing, however, is only slightly larger. The muted response of house prices reflects a relatively elastic demand for owner-occupied housing space by renters at this low price.

7.2.3 The Role of the Supply Shock

As many observers have noted, a portion of the decline in house prices most likely resulted from too many houses being built in the run-up to the crisis. If the supply shock were to be eliminated, house prices would decline 12.33 percent and the foreclosure rate would rise to 7.38 percent. At the margin, the supply shock accounts for 35 percent of the actual decline in prices and 40 percent of foreclosures. These marginal contributions confirm the intuition that overbuilding played a substantial role in the crisis.⁴⁶

The last line in Table 9 reports what happens when the only shock is overbuilding. The house prices drop 5.46 percent and foreclosures rise to 4.84 percent in the period of the shock. Although the effects are muted relative to the baseline, they are large compared with the (new) steady state: The house price drop overshoots its steady-state drop by 130 percent, and foreclosures in the period of the shock are 3.1 times their (new) steady-state value.

⁴⁶Note that since the model is nonlinear, the marginal effects of different factors can add up to more (or less) than the total effect.

8 Foreclosure Crisis and Policy

We use the model to predict how government actions might affect response to the shocks considered in this paper. We analyze two such policy actions. The first is a temporary increase in the inflation rate, such as might result from (temporarily) accommodative monetary policy in face of the crisis. The second is the elimination of the mortgage interest deduction; this action is in the spirit of “macroprudential policies” designed to make the economy less crisis-prone.

8.1 Unexpected Inflation and Foreclosures

We study the effects of a faster-than-expected inflation path on house prices and foreclosures. We assume that in the period of the shock, the anticipated inflation rate going forward rises to 4 percent for five years and then falls back to the steady state value of 2.5 percent. We assume that when the shock hits, the nominal interest rate at which the payment stream is evaluated is now (unexpectedly) higher – because anticipated inflation is higher.

The higher inflation path decreases the default rate from 13.71 percent to 7.04 percent. With a higher inflation rate, the real value of mortgage debt erodes more rapidly. Thus, the value of keeping the house is higher, and the value of selling the house is higher as well because the present discounted value of the outstanding loan to be repaid upon sale is now lower. For both reasons, fewer households find default as the best option. The lower default rate does not have much of an impact on house prices, which fall about 18.89 percent (as opposed to 19 percent).

8.2 Mortgage Deduction and the Scale of the Foreclosure Crisis

We study the effects of shocks when there is no tax incentive to take on leverage. As noted in the discussion of the steady state, eliminating the mortgage deduction lowers mortgage debt, and average home equity ratio is 78 percent. The reduction in leverage also reduces the steady-state foreclosure rate to 0.81 percent (about half of the foreclosure rate in the baseline model). If this less-leveraged economy is hit by the same set of shocks, house prices drop by 15.76 percent (roughly 3.2 percentage points less than in the baseline) and foreclosures rise to only 3.93 percent, about 10

percentage points lower than in the baseline model. Thus, the model predicts that the scale of the foreclosure crisis would have been much smaller if the tax code did not encourage leverage.

9 Conclusion

We presented a novel model of long-duration collateralized debt with endogenous down payment. We calibrated the model to match a small number of long-run facts regarding the U.S. housing and mortgage markets. We stressed that the federal tax code has important implications for these markets. The exemption of implicit rental income from taxable income is key for getting a large number of homeowners. The deductibility of mortgage interest payments from taxable income is key for getting people to borrow to purchase homes. Long duration of mortgage debt and inflation are important in producing the observed dispersed distribution of home equity.

We used the model to understand the foreclosure crisis. We showed that a modest level of over supply in the housing market, coupled with a plausible increase in the cost of new mortgages, can account for the steep decline in house prices. Given the decline in house prices, the model can account for much of the observed rise in foreclosures if we also consider the lengthening of the time to complete a foreclosure. With regard to the effects of policy parameters on crisis outcomes, two are worth noting. First, the scale of the crisis would have been much smaller if mortgage interest payments were not tax deductible, and, second, faster-than-expected inflation would have lowered the foreclosure rate.

10 Appendix on the Computational Algorithm

We start with a steady state without any aggregate shocks and then perturb the economy with a permanent unanticipated shock to the supply of housing and solve for the perfect foresight transition path to the new steady state.

10.1 Main Algorithm

The AR1 earnings process is approximated by a 17-state Markov chain.⁴⁷

The algorithm is as follows. We assume that the transition from the initial steady state to the new steady state takes 40 periods (years).

START OF OUTER LOOP

1. Guess a sequence of $z(t)$ and a sequence of $p(t)$ for periods 1 through 41. For $t = 1$, we normalize $z(1) = 1$ and set $p(1) = z(1)/(1 + \rho + \Delta - q)$.

START OF INNER LOOP

- (a) Guess value functions and mortgage pricing functions for period 1 and period 41. That is, guess $V_R(w, a, t)$, $V_R^D(w, a, t)$, $V_O(w, a, x, k, \delta_k, t)$, and $q(w, a, x, k, t)$ for $t = 1, 41$.
- (b) Solve for decision rules for $t = 41$, assuming that the value functions and the pricing function for $t = 42$ are the same as the guessed value and pricing functions for $t = 41$. This assumption imposes that we are in steady state in period 41. The decision rule for $t = 41$ implies new value functions and a new pricing function for $t = 41$. Replace the guessed value and pricing functions by these new value and pricing functions. Recompute the period 41 decision rule. Continue repeating this step until the new value and pricing functions are close to the guessed value and pricing functions.
- (c) Use the converged decision rule for $t = 41$ and the converged pricing function for period $t = 41$ to compute the pricing function for period $t = 40$ (see equation 7). Use this pricing function for $t = 40$ and the converged value function for $t = 41$ to compute the value function and decision rules for $t = 40$.
- (d) Proceed backward in this way, calculating new value functions, pricing functions, and decision rules all the way back to $t = 2$.
- (e) Solve for decision rules in $t = 1$ assuming that the value function and pricing function for $t = 2$ are the same as the guessed value and pricing functions $V_R(w, a, 1)$, $V_R^D(w, a, 1)$,

⁴⁷The choice of the number of grids for income was determined by the requirement that the properties of the solution not be sensitive to the number of grids. We found that grid size of 17 was sufficient for numerically stable results.

$V_O(w, a, x, k, \delta_k, 1)$, and $q(w, a, x, k, 1)$. Again, this imposes the assumption that we are in the steady state in period 1 (this is where we use the assumption that the shock that happens in period 2 is unanticipated). Update the price and value functions for $t = 1$ until they converge just as in step b.

END OF INNER LOOP

2. Use the converged decision rules for $t = 1$ to compute the initial steady-state distribution of people over the state space. Set the total owner-occupied housing space in period $t = 1$ to the total demand for owner-occupied housing space implied by the initial steady state distribution and the total supply of rental housing space to the total demand for rental housing space implied by the initial distribution.
3. Starting from this initial distribution in $t = 1$, use the decision rules computed for periods $t = 2, 3, \dots, 41$ to compute the distribution of households over the state space for periods $t = 2, 3, 4, \dots, 41$.
4. Use these distributions to compute excess demand for housing space in each of the years. The supply of owner-occupied housing space in periods 2 through 41 is simply (1.03) times the supply of owner-occupied housing space for $t = 1$ determined in step 2 and the supply of rental space in periods 2 through 41 is the supply of rental space for $t = 1$ determined in step 2.
5. For $t = 2, 3, \dots, 41$, update $z(t)$ and $p(t)$ appropriately (increasing the price slightly if there is an excess demand in that period and decreasing it slightly if there is an excess supply).
6. Repeat 1–5 until excess demand in each market in each period is almost zero.

END OF OUTER LOOP

Note that if the converged sequence of housing and rental prices does not change very much in the last several periods, that is a good indication that we are close to steady state by period 41.

10.2 Computation of Value Functions and Decision Rules

The value functions and decision rules are solved on a grid. The number of grid points for w is 17; for a it is 75; for x it is 80; and for k it is 15 (the depreciation shock δ has a two-point distribution). When solving for the decision rules for a' and x' , we allow for choices that are off the grid. In particular, we search over 15×75 points for a' and 5×80 points for x' . For values of a' and x' that are not on the grid, we use linear interpolation of the (future) value function (in effect, we assign them randomly to the relevant adjacent grid points).

To calculate the excess demand for owner-occupied and rental properties, we simulate the economy, keeping track of the measure of individuals on each grid point. For the simulations, we assume that if an individual chooses a' or x' off the grid, the individual is sent to the relevant adjacent grids according to the probabilities defined by the previously shown interpolation step.

To ensure continuity of the excess demand functions for the two types of housing with respect to the current and future prices embedded in the aggregate state variable t , it is generally necessary to allow for a small level of randomness in the discrete choices taken by households (for renters, these include the decision to buy or rent, and for homeowners, these include the decision to sell, keep, or default). For a homeowner, the probability of a given discrete action is given by

$$\text{Probability of } i = \frac{\exp(\nu K_i)}{\exp(\nu K_D) + \exp(\nu K_0) + \exp(\nu K_1)}, i = \{0, 1, D\} \quad (10)$$

and for a renter in good standing (analogous expressions hold for renters with bad credit) is given by

$$\text{Probability of } i = \frac{\exp(\nu M_i)}{\exp(\nu M_0) + \exp(\nu M_1)}, i = \{0, 1\}. \quad (11)$$

These expressions can be justified (McFadden (1974)) by appealing to the presence of an additive random perturbation to the payoff from each action, where each perturbation is drawn independently from a Type 1 Extreme Value distribution with scale parameter ν . The expressions imply that the discrete action with the highest payoff is always chosen with the highest probability, and this probability is close to 1 if the variance of the perturbations is low (see, for instance, Train

(2009), Chapter 3, Section 2).

Finally, we check the steady-state equilibrium for sensitivity to changes in the number of grid points.

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