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Abstract

An important inefficiency in sovereign debt markets is debt dilution, wherein sovereigns ignore the adverse impact of new debt on the value of existing debt and, consequently, borrow too much and default too frequently. A widely proposed remedy is the inclusion of seniority clause in sovereign debt contracts: Creditors who lent first have priority in any restructuring proceedings. We incorporate seniority in a quantitatively realistic model of sovereign debt and find that seniority is quite effective in mitigating the dilution problem. We also show theoretically that seniority cannot be fully effective unless the costs of debt restructuring are zero.

JEL: E44, F34, G12, G15

Key Words: Debt Dilution, Seniority, Sovereign Default

1 Introduction

Debt is one of the main vehicles through which firms and sovereign countries finance investment and consumption. While debt payments are typically noncontingent, default on debt is generally always a possibility, and we see both firms and countries defaulting frequently.

Given the deadweight costs of default, a major focus of the debt literature has been on understanding and proposing institutional arrangements that can make debt contracts more efficient so that the value of the firm or utility of the country is maximized. When long-term debt is involved, a major source of inefficiency is the *debt dilution problem*, wherein the borrower ignores the adverse impact on the value of outstanding debt when deciding whether to issue new debt and therefore borrows too much and defaults too frequently.[1] In the absence of an institutional arrangement that protects current long-term creditors from losses in value resulting from additional future borrowings, current creditors lend funds at an interest rate that is high enough to cover these anticipated losses (in expectation).[2] This makes long-term debt costly and ultimately creates welfare losses for the borrower.

The debt dilution problem has received considerable attention in policy debates surrounding sovereign debt crises. In particular, the possibility of debt dilution (and the attendant higher interest cost of debt) is thought to induce sovereign borrowers to opt for debt structures that are hard to dilute, such as very short-term debt (Sachs & Cohen (1982) and Kletzer (1984)) or debt that cannot be easily restructured so that the costs of default are high and the likelihood of default correspondingly low

[1]Argentina's decision to issue bonds to a wide base of borrowers in the late 1990s is an example of a sovereign choosing to dilute the value of debt in the hands of existing bondholders. Similarly, the decision by Russia and Ukraine to issue short-term debt in the months leading up to default in late 1998 is another example of dilution. In these instances, the possibility of dilution arguably encouraged these countries to take on more debt than they would otherwise and made default more likely.

[2]In practice, certain classes of creditors are generally (but not always!) accorded priority over others; for example, bonds generally get priority over bank loans. But within a particular class of creditors — the class of bondholders, for instance — treatment is generally equal.

(Shleifer (2002) and Dooley (2000)). But the tendency toward short-maturity debt exposes the sovereign to the risk of a confidence-driven rollover crisis (Giavazzi & Pagano (1990) and Cole & Kehoe (1996)), and the tendency toward hard-to-restructure debt makes crises very costly when they happen, perhaps inefficiently so (Bolton & Jeanne (2009)). The debt dilution problem has thus been viewed as an important reason for emerging-market borrowers to be crisis prone and, in the event of a crisis, experience a costly and protracted period of restructuring (Borensztein et al. (2006)).

In the last half-dozen years or so, a rapidly growing quantitative-theoretic literature has focused on explaining the unique characteristics of emerging-market business cycles (Neumeyer & Perri (2005), Aguiar & Gopinath (2006), and Arellano (2008), among others). This literature points to the highly volatile and strongly countercyclical country interest rates as a driver for the pronounced volatility of consumption and countercyclicality of net exports in (open) emerging economies. It builds on the negative relationship between default spreads and real output implicit in the model of defaultable sovereign debt developed in Eaton & Gersovitz (1981). Counterfactual exercises conducted with calibrated models that match debt-to-output ratios and typical maturity length of sovereign debt reveal that if the sovereign could issue debt whose value is protected from capital losses resulting from additional future borrowing, the frequency of default as well as the volatility of interest rates would decline substantially and aggregate welfare would improve (Chatterjee & Eyigungor (2012) and Hatchondo, Martinez & Sosa-Padilla (2011)). Thus, this literature on business cycles also points to debt dilution as the key reason why emerging economies borrow to the point where default spreads are large and volatile.

In the case in which creditors expect to recover something from the debtor in the event of default, the debt dilution problem can be mitigated if there is an explicit seniority structure on debt. This structure is based on the “first in time” or “absolute priority" rule that requires that creditors who lent first be paid in full before creditors who lent later are paid. Such a rule makes it harder to dilute the value of existing debt insofar as existing creditors do not have to share payment on the defaulted debt with new creditors.[3] Since a sovereign default is typically followed by a restructuring of the defaulted debt (wherein creditors are given new debt in place of old debt), imposition of a "first in time" rule has been proposed as a (partial) solution to the debt dilution problem (Borensztein et al. (2006), Bolton & Skeel (2004), and Bolton & Jeanne (2009)) as well as an aid to an orderly restructuring of debt following default (Gelpern (2004)).

The goal of this paper is to advance our understanding of the role of seniority in ameliorating the costs of debt dilution by incorporating seniority in a quantitatively realistic, infinite-horizon model of sovereign debt. The country borrows long-term in a competitive bond market and has the option to default on its debt. Following default, the sovereign bargains with its creditors to lower the level of debt (a debt write-down). Thus, there is payment on defaulted debt in the sense that the old debt is settled with new debt. During the period of default and renegotiation, the sovereign does not have access to international financial markets and suffers an output loss. Within this setup two different market arrangements are analyzed and compared. In the first, in the event of default, all creditors are treated equally (the current system); in the second, a "first in time rule" is followed.

We make three sets of contributions. First, on the methodological side, we show how the high dimensionality of the borrower's state space resulting from the need to keep track of the bond's time of issuance can be sidestepped by indexing each bond

[3] In the corporate finance literature, Fama & Miller (1972) gave an early discussion of how creditors of a firm can protect themselves from dilution by making their loans senior. It is important to note, however, that seniority is not a panacea. For instance, giving seniority to the most recently issued debt might lead to better outcomes if investment decisions are endogenous (Hennessey (2004)) or if there is the possibility of an inefficient default due to coordination difficulties among existing creditors (Saravia (2010)). Also, Bizer & DeMarzo (1992) show that if borrowers can influence the probability of default through their effort decisions, the ability to borrow sequentially can lead to inefficiently high levels of debt and default even if seniority among creditors is respected.

by its *rank* at the time of issuance. With this indexation scheme, a single additional continuous state variable is sufficient to keep track of seniority and thereby renders the model computationally tractable. The scheme is generally applicable and can be used to study the effects of seniority in other contexts such as corporate bonds.[4]

Second, on the quantitative side, we calibrate the model (in which all debts are treated equally in default) to Argentine facts: specifically, its average debt-to-GDP ratio, the average spread on its debt, and the level of repayment on its debt in the most recent (2001) default episode. Then, assuming no prior debt, we investigate how behavior of the model changes when we replace the current system with one where seniority is respected in default. We find that enforcing seniority reduces the frequency of default by 40 percent, reduces the average spreads by about 42 percent and increases the average debt carried by the economy slightly. The gain in welfare from these changes is around 1.0 percent of consumption in perpetuity. We also show that this gain is generally decreasing in the level of prior debt — if enforcing seniority requires making prior debt senior to any new debt.

Finally, on the theoretical side, we shed light on the limitations of seniority as a solution to the debt dilution problem. Specificially, we show that seniority cannot completely overcome the debt dilution problem unless settlement following default

[4] With regard to dimensionality, there are two separate challenges need to be met. First, to talk meaningfully about seniority one must allow for long-term debt. If it is assumed that the maximum maturity of any claim is T periods, in an ongoing dynamic setting one has to keep track of at least T *continuous* state variables (namely, the size of obligations that are due in the next T periods) to correctly compute default probabilities. Even for small values of T (say, 3 or 4), the problem becomes computationally intractable. One way to address this dimensionality challenge is to model long-term debt as debt that matures probabilistically as originally proposed in Leland (1998) and adopted in Chatterjee & Eyigungor (2012) (and others) and in this paper. If all bonds are treated equally in default, then there is no need to keep track of when a particular bond was issued as all bonds have the same payoff structure going forward. But if bonds that were issued earlier are given priority in default over bonds that were issued later, then it becomes necessary to keep track of when a particular bond was issued in order to determine its payoff in default. In the random maturity context, there may be bonds outstanding that were issued many, many periods ago. Once again, there are many continuous state variables (the quantity of surviving bonds issued at different times in the past) that need to be kept track of in order to compute prices. The indexation scheme proposed in this paper overcomes this problem by introducing a single state variable (the rank) that, in effect, keeps track of when a bond was issued and hence its priority in the event of default.

happens immediately and does not impose any resource costs on the sovereign. Since such costless renegotiation is very unlikely in practice (and perhaps undesirable as well) seniority will not solve the debt dilution problem fully in practice. Indeed, in our quantatitive analysis we find that it does not and the theoretical results help us understand why this is the case.

One question to address at the outset is whether respecting seniority requires additional commitment on the part of the sovereign. The answer is No. It is generally recognized (see for instance Borensztein et al. (2006)) that enforcing seniority requires giving senior creditors the legal basis to sue junior creditors who receive payments in contravention of their order of priority. Since the basis to sue must exist in the law of the country in which the bond is issued, a suitably fashioned seniority clause can be introduced into every bond contract at relatively low cost.[5] To strengthen the effectiveness of the clause, the clause may need to give senior creditors an "exit option" in case the sovereign issues new debt whose priority in relation to existing debt is unclear. For instance, the "exit option" could take the form of an immediate demand for repayment of debt whose seniority is potentially compromised by a new issuance. Thus seniority is an *inter-creditor* issue whose legal basis and required safeguards must exist in the courts of the country in which the debt is issued, not the court of the sovereign. In this sense, seniority does not involve any additional commitment on part of the sovereign.

[5] In recent years, the developed-country courts (where the sovereign debt is typically issued) have handed down rulings that suggest seniority in sovereign debt contracts will be enforced by the courts. See, for instance, the discussion in Bolton & Skeel (2005) (p.188-189) of the *Elliott Associates v. Government of Peru* case, wherein the court in Brussels permitted Elliott to compel EUROCLEAR — the entity responsible for disbursing payments to creditors who had accepted Peru's restructured debt in 2001 — to stop making these payments because Elliot's *prior claim* on the Government of Peru had not been satisfied. More recently, the court in New York handed creditors pursuing repayment on Argentina's 2001 defaulted debt similar power to stop payments to creditors who accepted Argentina's restructured debt in 2005.

2 Preferences and Endowments

Time is discrete and denoted $t \in \{0, 1, 2, ...\}$. The sovereign receives a strictly positive endowment y_t each period. The stochastic evolution of y_t is governed by a first-order finite-state Markov chain with state space $Y \subset \mathbb{R}_{++}$ and transition law $\Pr\{y_{t+1} = y'|y_t = y\} = F(y', y) > 0,\ y$ and $y' \in Y$.

The sovereign maximizes expected utility over consumption sequences, where the utility from any given sequence c_t is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),\ 0 < \beta < 1. \tag{1}$$

The momentary utility function $u(\cdot) : [0, \infty) \to \mathbb{R}$ is continuous, strictly increasing, and strictly concave.

3 Market Arrangements and the Option to Default

The sovereign can borrow in the international credit market with the option to default. We analyze long-term debt contracts that mature probabilistically (Hatchondo & Martinez (2009), Arellano & Ramanarayanan (2010), and Chatterjee & Eyigungor (2012)). Specifically, each unit of outstanding debt matures in the next period with probability λ. If the unit does not mature, which happens with probability $1 - \lambda$, it gives out a coupon payment z. We assume that unit bonds are infinitesimally small — meaning that if b unit bonds of type (z, λ) are outstanding at the start of the next period, the issuer's coupon obligations for the next period will be $z \cdot (1 - \lambda)b$ with certainty and the issuer's payment-of-principal obligations will be λb with certainty.

We assume there is a single type of bond (z, λ) available in this economy and that the sovereign chooses the size of its debt from a finite set $B = \{b_I, b_{I-1}, \ldots b_2, b_1, 0\}$, where $b_I < b_{I-1} < \ldots < b_2 < b_1 < 0$. As is customary in this literature, we will view debt as negative assets.

The option to default means that the sovereign has the right to unilaterally stop servicing its debt obligations — i.e., stop making coupon and principal payments. Default is costly in two ways. First, the sovereign loses access to the international credit market in the period of default — i.e., it cannot borrow or save internationally in the period of default. Second, the sovereign experiences a loss in ouput $\phi(y)$ in the period of default. These assumptions mean that in the period of default the sovereign consumes $y - \phi(y)$ units of goods. We assume that $y - \phi(y) > 0$ for all $y \in Y$ and that $y - \phi(y)$ is increasing in y.[6]

Since repayment on defaulted debt is a precondition for discussing the role of seniority, we incorporate a theory of repayment, but keep the theory relatively simple.[7] We assume that, in the period of default, the sovereign and its creditors bargain over the level of debt with which the sovereign will start the following period. The bargaining is assumed to be a one-shot Nash bargain with the bargaining power of the sovereign given by $\alpha \in [0, 1]$ (correspondingly, the power of the creditors is $(1 - \alpha)$). The bargaining game delivers a post-default debt level G, which depends on the level y in the period of default. We denote this schedule by $G(y)$.

Within this framework, we analyze two different market arrangements. In one arrangement, all creditors are treated equally in default. In the other arrangment, creditors are ranked by seniority, and junior claimants receive payments in default only if senior claimants have been fully compensated. For each market arrangement, we assume that lenders are risk-neutral and that the market for sovereign bonds is

[6]In this paper, a function $f(x)$ is *increasing (decreasing) in* x if $x' > x$ implies $f(x') \geq (\leq) f(x)$ and is *strictly increasing (strictly decreasing)* in x if $f(x') > (<) f(x)$.

[7]The theory of post-default bargaining between a sovereign and its creditors is an active area of research. Our approach is closest in spirit to Yue (2010). Benjamin & Wright (2009) and D'Erasmo (2012) provide alternative approaches that attempt to explain the delays in reaching settlement following default.

competitive.

4 The Model Without Seniority

4.1 Decision Problem of the Sovereign

With this market arrangement, the price of a unit bond will depend only on the current persistent component of output and on the level of outstanding debt. We will denote the price of a unit bond by $q(y, b')$.

Denote the lifetime utility of a sovereign that enters a period with (y, b) by $W(y, b)$. Then, the sovereign's payoff from default, denoted $X(y)$, is given by

$$X(y) = u(y - \phi(y)) + \beta E_{y'|y} W(y', G(y)), \tag{2}$$

where (as noted before) $G(y)$ is the debt level the sovereign agrees to in the post-default bargaining game (to be described later). The payoff from repaying the debt, denoted $V(y, b)$, is given by

$$\begin{aligned} V(y, b) &= \max_{b' \in B} u(c) + \beta E_{y'|y} W(y', b') \\ &\text{s.t.} \\ c &\leq y + (\lambda + [1 - \lambda]z)b + q(y, b')[(1 - \lambda)b - b']. \end{aligned} \tag{3}$$

The above assumes that the budget set under repayment is nonempty, meaning there is at least one choice of b' that leads to nonnegative consumption. But it is possible that all choices of b' may lead to negative consumption, in which case repayment is simply not an option. In this case the value of $V(y, b)$ is set to $-\infty$. In the event repayment is feasible, the optimal debt choice of the sovereign is denoted $a(y, b)$. We assume that if the sovereign is indifferent between two distinct b's it chooses the larger one (i.e., it chooses a lower debt level over a higher one).

Finally,

$$W(y,b) = \max\{V(y,b),\ X(y)\}. \tag{4}$$

Since W determines both X and V (via equations (2) and (3), respectively), equation (4) defines a Bellman equation in W. This equation implicitly determines the sovereign's default decision rule $d(y,b)$, where $d = 1$ if the default is optimal and 0 otherwise. We assume that if the sovereign is indifferent between repayment and default, it repays.

4.2 Equilibrium

The world one-period risk-free rate r_f is taken as exogenous. Under competition, the price of a unit bond satisfies the following pricing equation:

$$q(y,b') = \tag{5}$$
$$E_{\{y'|y\}}\left[[1-d(y',b')]\frac{\lambda+(1-\lambda)[z+q(y',a(y',b'))]}{1+r_f} + d(y',b')\frac{G(y')}{b'}\frac{q(y',G(y'))}{1+r_f}\right].$$

Note that in the event of default, the sovereign will agree to a debt settlement whose aggregate value is $q(y',G(y'))G(y')$. Since the total amount of debt defaulted on is b', each unit of defaulted debt will receive $q(y',G(y'))G(y')/b'$. One can interpret the term $G(y')/b'$ as the recovery rate and $[1-G(y')/b']$ as the "haircut" suffered by lenders in the event of default.

It is worth noting that the combination of repayment on defaulted debt and the possibility of debt dilution can lead to behavior that blurs the distinction between repayment and default. When default is imminent, the sovereign may choose to borrow as much as it can and then default *with certainty*. To understand why, suppose that the sovereign services its current debt b and then issues enough debt, denoted M', so that default is certain in next period. What does it gain from this strategy? The sovereign has to service the outstanding debt, which costs $[\lambda + (1-\lambda)z]\, b$. Offsetting this cost is the revenue that comes from new issuances of debt, which is equal to $-q(y, M')\,(M' - [1-\lambda]b)$. Since default is certain for M', the aggregate value of its obligations in the next period will be $q(y', G(y')) \times G(y')$. Therefore, $q(y, M')M' = E_{y'|y}\left[\frac{q(y',G(y'))\times G(y')}{1+r_f}\right]$, which is independent of M'. Therefore, the sovereign can maximize revenue from the new issuances by minimizing $q(y, M')(1-\lambda)(-b)$. Now observe that, for any $b' \leq M'$, $q(y, b') = E_{y'|y}\left[\frac{q(y',G(y'))}{1+r_f} \times \frac{G(y')}{b'}\right]$. Hence, $q(y, M')(1-\lambda)(-b)$ can be minimized by making M' go to -∞, which makes the price of existing debt go to zero. So whenever $\Delta = E_{y'|y}\left[\frac{q(y',G(y'))(-G(y'))}{1+r_f}\right] - [\lambda + (1-\lambda)z]\,(-b)$ is positive, default may be preceded by maximum dilution.[8] By borrowing as much as it can today, the sovereign dilutes the value of existing debt as much as possible and thereby promises as much of its default payment as possible to investors who buy its new debt. In this way, the sovereign increases its current consumption at the expense of its existing creditors and postpones the costs of default by one period.

Issuing as much debt as possible before default is not optimal if the debt is for one period ($\lambda = 1$) or if there is no repayment following default ($G(y) = 0$). In the first case, there is no outstanding debt to dilute and, in the second case, borrowing beyond the point where default is certain means that $q(y, b')b'$ is zero. In either case, the sovereign does not get any extra consumption by this strategy.[9]

[8] The payoff from defaulting in the current period is $X(y)$, while the payoff from issuing an infinite amount of debt is $u(y + \Delta) + \beta E_{y'|y} X(y')$. Since the y process is persistent (i.e., $F(y, y)$ is close to 1), if $\Delta > 0$ then defaulting on an infinite amount of debt will generally be better than defaulting on a finite amount of debt.

[9] In Yue (2010), debt recovery after default is of the form $G(y')$, but there is no gain to issuing huge amounts of debt prior to default because debt is short-term. Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009) have long-term debt, but there is no repayment following default, so there is no incentive to dilute existing debt prior to default. More generally, this strategy of borrowing as much as possible prior to default is not optimal if the Δ defined in the text negative. This will happen if $(-b)$ is too high or if $E_{y'|y}\left[\frac{q(y',G(y'))(-G(y'))}{1+r_f}\right]$ is too low, i.e., if the gain from dilution is not large enough to more than offset the current debt service payments.

5 The Model with Seniority

The straightforward way to keep track of seniority is to introduce the "time since issuance" as an additional state variable. But, given the random maturity nature of our bonds, this would require keeping track of many (potentially, infinitely many) continuous states. To circumvent this problem, we propose a computationally tractable way of keeping track of seniority that does not involve keeping track of the time since issuance of any particular claim.

The idea is to assume that every unit bond outstanding has a rank denoted by s. If there are b units of the debt outstanding, the rank of any given unit bond is a unique number $s \in [b, 0]$. The closer s is to 0, the higher is the rank of the unit bond and (as will become clear below) the higher is its seniority in the event of default. We continue to assume that b is a member of the finite set B. However, s is a *continuous* variable since it is a member of $[b, 0]$. In effect, we are assuming that the sovereign can issue bonds in chunks $\{b_1, b_2, \ldots, b_I\}$ but within each chunk there is a ranking of unit bonds that compose that chunk.

How does the rank of a unit bond evolve over time? Suppose that the sovereign has b unit bonds outstanding at the end of any period. Consider a unit bond with rank $s \in [b, 0]$. Since any unit bond has a probability λ of maturing next period, among the bonds that are higher ranked than s there is a fraction λ that will mature. Thus, at the start of the next period, there will only be $(1 - \lambda)s$ bonds with a rank higher than s. This means that we can preserve the current ranking among all bonds with a rank greater than or equal to s if we reset the rank of each unit bond that survives into the next period to $(1 - \lambda)s$. This resetting rule implies that the unit bond with rank 0 (the most senior unit bond) continues to have rank 0 as long as it survives and any other unit bond's rank approaches 0 at a geometric rate the longer it survives.

If the sovereign issues new bonds in any period (i.e., $b' < (1-\lambda)b$), each member of the mass of newly issued bonds is assigned a unique rank s in the (semi-open) interval $[b', (1-\lambda)b)$. If the sovereign buys back debt (i.e., $b' > (1-\lambda)b$), then it is safe to assume that it is the mass of the most junior bonds — namely, the unit bonds with $s \in [(1-\lambda)b, b']$ — that are bought back (this point is discussed in more detail below).

Since the payoff to the bondholder in the case of default depends on the rank (seniority) of the bond held, the price of a unit bond will now depend on its ranking. Denote the price of a unit bond with $s \geq b$ by $q(y, b, s)$. With some abuse of notation, we will continue to use the same notation for lifetime utility $W(y, b)$ (and other value functions and decision rules), although there is no presumption that these functions are the same as in the case without seniority. Then, the payoff from default, $X(y)$, is given as in (2). The payoff from repayment is given by

$$V(y,b) = \max_{b' \in B} u\left(c\right) + \beta E_{(y'\, m')|y} W(y', b') \tag{6}$$
$$\text{s.t.}$$
$$c = y + m + \left[\lambda + (1-\lambda)z\right] b + R(y, b, b'),$$

where $R(y, b, b')$ denotes the revenue received from changing the asset level from b to b' (it will be positive if new bonds are issued and negative if bonds are bought back). This function is given by

$$R(y,b,b') = \begin{cases} q(y,b',b')\left[(1-\lambda)\,b - b'\right] & \text{if } (1-\lambda)\,b \leq b' \\ \int_{b'}^{b(1-\lambda)} q(y,b',s)ds & \text{if } (1-\lambda)\,b > b'. \end{cases}$$

In the event of a buyback, the sovereign buys the least senior bonds at a price that is equal to the price of the junior-most bond *after* the buyback. The reason for this specification is as follows. First, whether the sovereign buys senior or junior bonds will have no effect on the seniority structure of oustanding debt, since junior debt will rise in seniority if senior debt is bought back. Second, as we show later in the paper, the price function $q(y, b', s)$ is increasing in s (more senior bonds trade at a higher price). The combination means that the sovereign will minimize its purchase cost if it purchases junior-most debt. Finally, observe that a bondholder with a bond with $s > b'$ knows that he will be in possession of a bond whose value following the buyback will be $q(y, b', b')$ and therefore will be unwilling to sell his bond at any price less than $q(y, b', b')$. The upshot is that a sovereign that wishes to buy back debt can implement its plans at the least cost if it announces that it will buy debt back at the price $q(y, b', b')$.[10] In the event of new issuances, the revenue obtained from the sale is simply the sum (more precisely, the integral) of the price of each new unit sold.

As before, $W(y, b)$ is given by (4).

5.1 Equilibrium

With seniority, the market price of a bond will depend on its rank because of the role of seniority in determining payment in default. In the event of default, the bearer of a bond with seniority s will be compensated with new debt only if $s \geq G$, where G is the amount of new debt agreed to in the renegotiation. Then, price of a unit bond of rank $s \geq b'$ satisfies

$$q(y, b', s) = E_{y'|y}\left[[1 - d(y', b')]\frac{\lambda + [1-\lambda][z + q(y', a(y', b'), \max([1-\lambda]s, a(y', b')))]}{1 + r_f}\right] + E_{y'|y}\left[d(y', b')\frac{\mathbb{1}_{\{s \geq G(y')\}} q(y', G(y'), s)}{1 + r_f}\right], \tag{7}$$

where $\mathbb{1}_{\{\cdot\}}$ is an indicator function that takes the value 1 if the expression in $\{\cdot\}$ is true and 0 otherwise. Thus, a bond will receive payment in default only if its seniority makes it part of the new settlement. Observe that in the event there is

[10]Bondholders who hold unit bonds with ranking $s > b'$ have bonds whose price, namely, $q(y, b', s)$, is at least as large as $q(y, b', b')$ and therefore will have no incentive to participate in the buyback.

repayment (and the bond does not mature) the rank of bond becomes $(1-\lambda)s$ if the bond is not bought back, i.e., $a(y', b') \leq (1-\lambda)s$. If the bond is bought back, the price the bondholder will receive will be equal to the price of a unit bond with rank $a(y', b')$, as explained above.

We give a characterization result regarding the behavior of the equilibrium price schedule with respect to s and confirm our earlier claim that, all else remaining the same, the price of a unit bond is increasing in seniority (or rank) s. The proof of this claim essentially follows from the observation that the payoff to a bond of rank s cannot be any less (under any state of the world) than the payoff to a bond of rank $s' < s$.

Proposition 1 *For any $G(y)$, the equilibrium price function $q^*(y, b', s)$ is increasing in s.*

6 Post-Default Bargaining Game

In this section, we describe how the function $G(y)$ is determined for the two market arrangements. As noted earlier, we assume that, in the period of default, creditors and the sovereign Nash-bargain over the new level of debt that the sovereign will start servicing in the period following default. We assume that the threat points of this one-shot bargaining game are as follows: If there is no agreement, the creditors receive nothing, i.e., there is never any repayment of any kind on the debt. Correspondingly, if there is no agreement, the sovereign permanently loses access to international capital markets. Let $A(y)$ denote the lifetime utility of being in autarky when current endowment is y. Then, $A(y)$ solves

$$A(y) = u(y) + \beta E_{y'|y} A(y'). \tag{8}$$

In the case where seniority is not enforced, $G(y)$ solves

$$\max_{\{G\in B\}} \left[E_{y'|y}\left[W(y',G)-A(y')\right]\right]^{\alpha}\left[q(y,G)(-G)\right]^{1-\alpha} \tag{9}$$

s.t.

$$E_{y'|y}[W(y',G)-A(y')] \geq 0 \text{ and } q(y,G)(-G) \geq 0.$$

All creditors would wish to push down G all the way to $G^*(y)$, the level of debt for which $q(y,G)(-G)$ is maximized. If this level of G is consistent with a positive surplus for the sovereign and if the sovereign's outside option is very poor or if its bargaining power is very low, Nash bargaining may well lead to $G(y) = G^*(y)$. More generally, the bargaining will lead to some $0 \geq G(y) \geq G^*(y)$.[11]

When seniority is enforced, the fact that all bonds are not treated equally in default can lead to conflict among creditors. In particular, a holder of a junior bond has an incentive to push for a low value of G in an effort to get the bond included in the settlement, while a (senior) bondholder whose bond is already part of the settlement might suffer a loss in value as G is lowered. However, if every creditor owns an equal fraction of bonds in any seniority segment $(s, s+\Delta)$, it is in the interest of every creditor to increase the aggregate value of bonds issued in the settlement as much as possible.[12] Under this assumption, we can continue to

[11]Strictly speaking, the creditors cannot ask the sovereign to agree to a debt level that is significantly more than the face value of the debt defaulted on. Thus, one might reasonably include a constraint of the form $G \geq k(1+r_f)$, where k is the level of debt in default. However, because default generally occurs when the sovereign has a strong incentive to reduce the burden of debt, this will usually not be a binding constraint.

[12]While this is not likely to hold in practice, it may be in the interests of all creditors to operate as if it holds. In other words, it may be in their interest to agree to a bargaining protocol in which the negotiation proceeds under a "veil of ignorance" regarding the distribution of bonds across creditors. Such a protocol would reduce uncertainty regarding payouts in the event of default and thereby enhance the liquidity of such bonds which would benefit all creditors.

assume that $G(y)$ solves

$$\max_{\{G \in B\}} \left[E_{y'|y} W(y', G) - E_{y'|y} A(y')\right]^{\alpha} \left[\int_G^0 q(y, G, s) ds\right]^{1-\alpha} \tag{10}$$

$$\text{s.t.}$$

$$E_{y'|y}[W(y', G) - A(y')] \geq 0 \text{ and } \int_G^0 q(y, G, s) ds \geq 0.$$

An alternative bargaining protocol that might seem natural in this context, and which does not depend on the distribution of bonds across creditors, is one where a bond of seniority G is included in the settlement, provided the owner of that bond compensates the owners of bonds with seniority $s > G$ for the loss incurred as a result of G's inclusion. Such a protocol would be in the same spirit as covenants — common in corporate debt — that prohibit any payments to junior creditors that strip value away from senior creditors.

Under this protocol there will be an incentive on the part of the owner of a bond with seniority G to include it in the settlement if and only if $q(y, G, G) + \int_G^0 \partial q(y, G, s)/\partial G \, ds < 0$. The first term is the market price of the bond and the second term is the loss in value of all senior bonds as a result of G's inclusion in the settlement. But this is exactly the condition that $\int_G^0 q(y, G, s) ds$ should be decreasing in G , i.e., the condition that the integral should increase as G is lowered.[13] Under this protocol, therefore, the value of $G(y)$ will also be determined

[13] For ease of exposition, we are treating G as a continuous variable. If we were to respect the fact that G is being chosen from a discrete set, we would say that a bond of seniority s qualifies for the settlement provided the junior-most bond in the "chunk" to which s belongs qualifies for the settlement. More formally, let $G^+(s)$ and $G^-(s)$ be the senior-most and junior-most bonds in the chunk to which s belongs. Then, s qualifies for settlement if and only if

$$\frac{\left[\int_{G^+(s)}^0 q(y, G^-(s), z) dz - \int_{G^+(s)}^0 q(y, G^+(s), z) dz\right]}{G^+ - G^-} + q(y, G^-(s), G^-(s)) \geq 0.$$

The first term is the loss in value to senior bondholders from inclusion of the chunk of bonds containing s scaled by the size of the chunk, and the second term is the value of the junior-most bond in the chunk containing s. The scaling of the loss implies that it is being spread equally among all bonds in the chunk containing s.

exactly as in (10), although the interpretation of the solution would be different. If one imagines the bonds lined up by seniority, the bargaining issue is where to draw the line in terms of inclusion in the settlement. The first requirement, which pertains to the distribution of the default payment between creditors, is one of *efficiency*: A bond of seniority s qualifies for the settlement provided the aggregate value of the settlement is not any lower as a result of its inclusion. The second requirement, which pertains to the distribution of the surplus between creditors and the sovereign, is one of *welfare*: A qualifying bond is included in the settlement provided the Nash product is higher as a result of its inclusion.

Although $G(y)$ is determined in the same way under both bargaining protocols, the solution will differ because the second bargaining protocol will alter the bond price function. In particular, the bond price equation will now satisfy the modified equation:

$$\begin{aligned} q(y,b',s) = & \\ E_{y'|y} & \left[[1-d(y',b')] \frac{\lambda + [1-\lambda][z + q(y', a(y',b'), \max([1-\lambda]s, a(y',b')))]}{1+r_f} \right] \\ & + E_{y'|y} \left[d(y',b') \frac{\mathbb{1}_{\{s \geq G(y')\}} \left[q(y',s,s) + \int_s^0 \partial q(y,s,z)/\partial G \, dz \right]}{1+r_f} \right]. \end{aligned} \tag{11}$$

Observe that, in the event of default, a holder of a bond of seniority s, provided it is part of the settlement ($s \geq G(y')$), receives $q(y',s,s)$ adjusted downward by the compensation paid out to protect the value of senior bonds.[14]

In the quantitative analysis, we experimented with both bargaining protocols. It turns out that, for our calibration of the model, the two protocols give virtually the same answers. The reason is that when the first bargaining protocol is followed, the

[14]When discreteness in G is respected, the bondholder of seniority s receives

$$q(y, G^-(s), s) + \frac{\left[\int_{G^+(s)}^0 q(y, G^-(s), z) dz - \int_{G^+(s)}^0 q(y, G^+(s), z) dz \right]}{G^+ - G^-}.$$

negative effect on the value of senior bonds as G is lowered is very small for a range of G values around $G(y)$. Consequently, $G(y)$ (and other equilibrium quantities) changes very little if the second bargaining protocol is followed. In the rest of the paper, we follow the first protocol since it is easier to compute.

7 Seniority and Welfare: The Argentine Case

In this section we explore the quantitative implications, both positive and normative, of enforcing seniority. We focus on Argentina, the country most intensively studied in the quantitative sovereign debt literature. To make the model quantitative, we assume that $u(c)$ is a CRRA function with curvature parameter $(1-\gamma)$ and that $\ln(y)$ follows an AR1 process with parameters $(\rho, \sigma_\epsilon^2)$. Following Arellano (2008), we assume that the form of the default cost function $\phi(y)$ is given by $\max\{0, y - y_D\}$, where $y_D > 0$ is some constant.

7.1 Calibration

The value of γ is set at 2, which is the standard value used in this literature. The parameters of the output process are estimated on linearly detrended quarterly real GDP data for the period 1980:1 to 2001:4 under the assumption that real GDP is a sum of the persistent AR1 process and a purely transitory shock with a small variance.[15] The estimated values of ρ and σ^2 are 0.948503 and 0.027092^2, respectively. The risk-free rate r_f is set to 0.01, which corresponds to an annual rate

[15]The transitory shock is needed to avoid convergence issues at the model solution stage (see Chatterjee & Eyigungor (2012) for details). We include the transitory shock in the estimation of (ρ, σ^2) to ensure that the overall variability of GDP assumed in the model is the same as in the data. The variance of the transitory shock is set at 0.004^2, which is less than 1 percent of the variance of de-trended GDP over this period. The quarterly data series on real GDP and the (nominal) yield on Argentine sovereign debt are taken from Neumeyer & Perri (2005). The GDP data were deseasonalized using the multiplicative X-12 routine in EViews.

Table 1: Parameters Selected Independently

Parameter	Description	Value
γ	Risk Aversion	2
σ	Standard Deviation of Output Innovation	0.027092
ρ	Autocorrelation of Output Process	0.948503
r_f	Risk-free Return	0.01
λ	Reciprocal of Avg. Maturity	0.05
z	Coupon Payments	0.03

of 4.0 percent.[16] The parameters describing the bond were determined to match the maturity and coupon information for Argentina reported in Broner, Lorenzoni & Schmukler (2007). The median maturity of Argentine bonds is 20 quarters so $\lambda = 1/20 = 0.05$. We set $z = 0.03$, corresponding to an annual coupon rate of 12 percent.[17] These selections are summarized in Table 1.

The three remaining parameters, namely, β, y_D, and α, are determined jointly to match three target statistics. The first statistic is the average external-debt-to-output ratio for Argentina over the period 1993Q1 to 2001:Q4, which is 1.0.[18] The model analog of this ratio is b/y.[19] The second statistic is the average annualized

[16]This is roughly the average nominal yield on 3-month U.S. Treasury bills over the period 1980:1 to 2001:4. The T-bill rate series used is the TB3MS series available at `http://research.stlouisfed.org/fred2/categories/116.`

[17]In the data, the value-weighted average coupon rate is about 11 percent. We chose 12 percent because, with an annual risk-free rate of 4 percent and an average spread of around 8 percent, a bond with a coupon of 12 percent will trade roughly at par. So, whether the debt is recorded at face value (which is the accounting practice) or at market prices (which is economically more sensible) will not matter for the calibration of the model.

[18]Debt is total long-term public and publicly guaranteed external debt outstanding that is disbursed and owed to private and official creditors at the end of each year, as reported in the World Bank's Global Development Finance Database (series DT.DOD.DPPG.CD).

[19]In the GDF database, the external commitments of a country are reported on a cash-accounting basis, which means that commitments are recorded at their face value, i.e., they are recorded as the undiscounted sum of future promised payments of principal (see "Coverage and Accounting Rules" in Section 3 of the World Bank Statistical Manual on External Debt, also available at http://go.worldbank.org/6FB4093970). The coupon payments agreed to do not figure directly in this accounting because they are not viewed as obligations until they are past due. Given this valuation principle, the model analog of debt as reported in the data is simply b. To see this, observe that each bond can be viewed as a combination of unit bonds with varying maturities. For instance, a measure λ of unit bonds is due in the next period, a measure $(1-\lambda)\lambda$ is due in two periods, ..., a measure $(1-\lambda)^{j-1}\lambda$ is due in j periods, and so on. Since each of these obligations has a face value of 1, each would be recorded as a unit obligation. Thus, the total obligation is

spread on Argentine sovereign debt over the same period, which is 0.0815.[20] The third statistic is the average recovery rate in the event of default. The model analog of this statistic is $G(y')/b'$ averaged over default episodes. Since defaults are relatively rare events, we target the recovery rate observed in the most recent Argentine default, which is 0.30.

While straightforward in principle, this moment-matching step is made challenging by the fact that, in our model, default appears to be generally preceded by maximum dilution (the logic for why this happens was explained in section 4.2). Since we do not observe maximum dilution in reality, there is presumably some real-world force that prevents this sort of behavior. One plausible force is *underwriting standards* that, in effect, impose an upper bound on the anticipated probability of immediate default on a new issue of bonds.[21] In what follows, we incorporate this force in our model in the form of a constraint requiring that, if $[(1-\lambda)b-b'] > 0$, then $E_{y'|y}d(y',b')$ cannot exceed some $\delta \in (0,1)$. For the baseline calibration, $\delta = 0.75$. Since δ is the probability of default in the next quarter, the implied annualized default probability is $1-(0.25)^4 = 0.9961$, which is only slightly less than 1. Therefore the constraint prohibits issuing new debt on which default is almost certain within the next year. We show later in the paper that there is a range of δ values below 0.75 for which our results remain unaffected. Thus the specific choice of δ is not crucial for the findings of the paper.

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simply $\sum_{j=1} \lambda(1-\lambda)^{j-1} = 1$.

[20]The default spread in the model is calculated as in the data. We compute an internal rate of return $r(y,b')$, which makes the present discounted value of the promised sequence of future payments on a unit bond equal to the unit price, that is, $q(y,b') = [\lambda + (1-\lambda)z]/[\lambda + r(y,b')]$. The difference between $(1+r(y,b'))^4 - 1$ and $(1+r_f)^4 - 1$ is the annualized default spread in the model. If there is no possibility of default, the unit price would be a constant $\bar{q}$ such that $\bar{q} = [\lambda + (1-\lambda)(z+\bar{q})]/[1+r_f]$, which implies $\bar{q} = [\lambda + (1-\lambda)z]/[\lambda + r_f]$. Since $q(y,b') \leq \bar{q}$, it follows that $r(y,b') \geq r_f$. Furthermore, the higher the probability of default, the lower is $q(y,b')$ and the higher is $r(y,b')$.

[21]Sovereign bonds issued in financial centers (such as New York, London, Frankfurt, and Tokyo) have to be underwritten by some investment bank, and reputational concerns may make these banks wary of issuing bonds on which the probability of immediate default is very high. Indeed, as noted in Flandreau et al. (2009), such standards have been common in sovereign debt markets for a long time, although the "gatekeeping" role appears to have shifted away from investment banks toward credit rating agencies in recent times.

Table 2: Parameters Selected Jointly

Parameter	Description	Value
y_D	Default Cost Parameter	0.53
β	Discount Factor	0.9374
α	Sovereign's Bargaining Power	0.42

Table 2 reports the parameter values that jointly come close to matching the three targets, given $\delta = 0.75$. Although all three parameters affect these statistics to some extent, the default cost parameter is key to matching the debt-to-output ratio; the discount factor is key for matching the average spread (through its effect on default frequency); and the bargaining power of the sovereign is key to matching the recovery rate.

Two statistics that we do not target but are important in the model are the default frequency and the volatility of spreads. These statistics and their model counterparts are reported in Table 3, along with targeted statistics and model counterparts. We give a range for the default frequency in the data because the estimate of this frequency varies with the sample period considered.[22] Our calibration implies a default frequency that is at the high end of the range reported and a volatility of spreads that is somewhat lower than in the data.

Table 3: Targeted and Untargeted Statistics: Data and Model

Statistics	Data	Model
Average Debt-to-Output Ratio	1.0	1.01
Average Annualized Spread	0.0815	0.0809
Recovery Rate	0.30	0.30
Annualized Default Frequency	$0.086 - 0.03$	0.085
S.D. of Annualized Spreads	0.0443	0.0380

[22] The frequency of default is the number of default episodes as a fraction of the number of years Argentina was in good standing with international creditors and therefore had access to international credit markets. If we look at the period 1946-2001 and use the "years in restructuring" reported in Beim & Calomiris (2000) (Table A) as the number of years Argentina was *not* in good standing, Argentina would show three defaults in a 35-year period of good standing. This would give a default frequency of 0.086. If we focus on 1800-2001 and use Beim and Calomiris again, the default frequency would drop to 0.03.

7.2 Welfare Gain from Enforcing Seniority

Table 4: Welfare, Debt, and Default with and without Seniority

Statistic	w/o Seniority	w/ Seniority
Welfare in Consumption Equivalent ($b = 0$)	1.0282	1.0395
Average Annualized Spread	0.0809	0.0466
Std. Dev of Annualized Spreads	0.0380	0.0236
Annualized Default Probability	0.0853	0.0512
Recovery Rate	0.30	0.38
Average Debt-to-Output Ratio	1.00	1.03

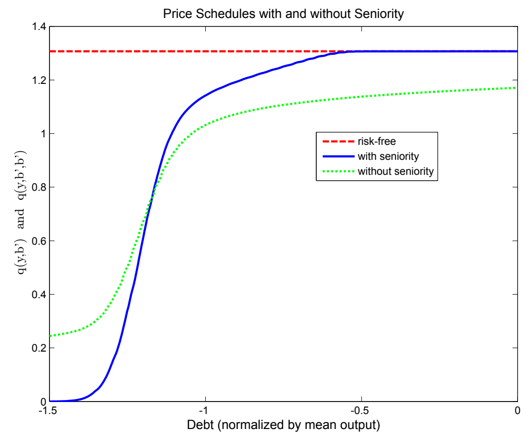
Table 4 reports the effects of moving from an environment without seniority to one in which there is seniority when the initial debt level b is zero. There is a significant increase in the lifetime utility of the sovereign from this move. In the absence of seniority, the value of constant consumption that gives the same average lifetime utility starting from zero debt is 1.0282, where the average is computed over the invariant distribution of y.[23] If the sovereign were to issue bonds that respect seniority, the constant consumption equivalent of the new average lifetime utility, similarly computed, would be 1.0395. Thus, in constant consumption equivalents, the sovereign is willing to pay an additional 1.0981 percent of consumption in perpetuity for this arrangement.

The source of the welfare gain can be understood by examining how seniority alters the bond price schedule. Figure 1 displays the price schedules for the two market arrangements along with the price of risk-free debt, indicated by the horizontal line.[24] Focusing first on the price schedule when seniority is not enforced (the dotted line), observe that there is a spread on bonds even for low levels of debt. This spread reflects lenders' expectation that even if the sovereign issues very little debt today, it will have the incentive to rapidly accumulate debt in the future so that the likelihood of default on some portion of the debt issued today is strictly positive.

[23] More precisely, the value for consumption reported is the value for which $c^{1-\gamma}/(1-\gamma) = (1-\beta)\sum_y W(0,y)\Phi(y)$, where $\Phi(y)$ is the invariant distribution of the transition law $F(y,y')$.

[24] The risk-free price $\bar{q}$ solves the equation $\bar{q} = (1+r_f)^{-1}[\lambda + (1-\lambda)z\bar{q}]$.

Figure 1: The Impact of Seniority on Bond Prices



Furthermore, the spread widens as more debt is issued because the expected "time to default" shrinks.

In striking contrast, when seniority is enforced, the sovereign can borrow at essentially the risk-free price up to a substantial level of debt. For a range of debt levels, bondholders are almost fully protected in default: Their seniority is high enough that, in the event of default, they are always part of the settlement and their loss is only the (missed) coupon payment in the period of default. For higher levels of debt there are two countervailing forces at work: On the one hand, if there is default and the bond is not part of the settlement, then the payoff in default is zero. This is a much bigger loss than when seniority is not enforced because, in the latter case, every bondholder shares in the default payment. By itself, this would cause the price of debt to be lower when seniority is enforced. On the other hand, as time passes without default and the bond does not mature, the rising seniority of the bond will eventually place it in a "protected zone" and the loss in the event of default will be minimal. By itself, this is a force that makes the price of debt higher than when seniority is not enforced. As we go beyond the initial "risk-free" range, the second effect is dominant and the price of debt continues to be higher than when seniority is not enforced. Eventually, though, the first effect dominates and the price of debt under seniority is actually lower than when seniority is not enforced.

It is evident from Figure 1 that, for each y, there is a range of debt levels for which the sovereign can borrow at a lower interest rate if seniority is enforced than when it is not. Consequently, the average equilibrium spread on sovereign debt is considerably lower when seniority is enforced, as noted in Table 4. A portion of the welfare gain from enforcing seniority results from this reduction in interest rates: Since the sovereign accesses the capital market as a borrower, it enjoys a positive wealth effect from lower borrowing costs.

The second important source of the welfare gain seen in Table 4 is the decline in the frequency of default. Because default is costly in terms of output, a lower default frequency raises average consumption. To understand why the frequency of default is lower when seniority is enforced, we need to understand the incentives that the sovereign faces to extend its borrowing into regions where the probability of default is higher. Suppose that the sovereign issues additional debt in the current period, i.e., $b' < (1-\lambda)b$. In the absence of seniority, the revenue from a marginal unit of debt sold is (treating b' as a continuous variable)

$$q(y, b') + \frac{\partial q(y, b')}{\partial b'}[b' - (1-\lambda)b].$$

The term $q(y, b')$ is the revenue from the sale of the marginal unit. But since $q(y, b')$ is increasing in b', this sale decreases the value of all bonds (the ones being currently issued as well as the ones that already exist) by $\frac{\partial q(y,b')}{\partial b'}$. This means that the sovereign loses the amount $\frac{\partial q(y,b')}{\partial b'}[b' - (1-\lambda)b]$ on all the inframarginal sales. The important point to note here is that although the sale of the marginal unit reduces the value of all outstanding debt, the sovereign cares only about the loss on the inframarginal sales, i.e., the loss imposed on the *new* debt being issued. In effect, the marginal sale expropriates resources from existing creditors in favor of new creditors by the amount

$$\frac{\partial q(y,b')}{\partial b'}(1-\lambda)(-b). \tag{12}$$

This expropriation allows the sovereign to sell new debt at a higher price than if this expropriation were not permitted.[25]

When seniority is enforced, and assuming again that $b' < (1-\lambda)b$, the revenue gain from the marginal unit of debt sold is

$$q(y,b',b') + \int_{b'}^{b(1-\lambda)} \frac{\partial q(y,b',s)}{\partial b'} ds.$$

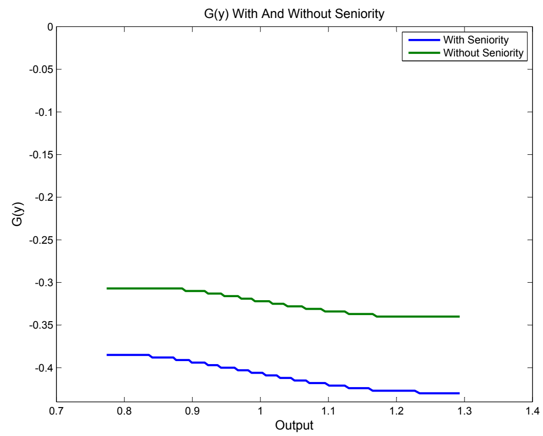
The sovereign receives $q(y,b',b')$ from the sale of the marginal unit, but loses some revenue on its inframarginal sales because of depreciation of the value of the inframarginal units. Since the price of bonds with different rankings will be affected by different amounts from the increase in debt, the loss is the integral term in the expression above. With seniority, the expropriation term analogous to (12) is

$$\int_{b(1-\lambda)}^{0} \frac{\partial q(y,b',s)}{\partial b'} ds.$$

This is the decline in the value of outstanding debt caused by the sale of the marginal unit of debt. Since this debt is senior, its value falls much less with new issuances of debt (why it falls at all when seniority is enforced is discussed in more detail in the last section of the paper). Since less is being expropriated from existing creditors, the price of the marginal bond is more sensitive to new issuances of debt. This effect is clearly evident in Figure 1: For high levels of debt, the $q(y,b',b')$ function

[25]Existing quantitative models of long-term sovereign debt and default assume no repayment in default. In this case, the expropriation results from the loss in the discounted value of coupon payments promised to existing creditors as a result of the higher probability of default. When there is some repayment in default, as is the case in our model, then expropriation also takes the form of the default payment being shared among a larger number of claimants.

Figure 2: Impact of Seniority on Resettlement



falls more steeply than the $q(y, b')$ function. The enhanced sensitivity of the price of debt to new issuances dissuades the sovereign from extending borrowing into regions where probability of future default is significantly positive and accounts for the drop in the frequency of default when seniority is imposed.

Aside from the two effects that have a strong bearing on the welfare gain, Table 4 shows other changes that are noteworthy. First, enforcing seniority raises the recovery rate in the event of default, from 0.30 to 0.38. The increase results from the higher value of capital market access for the sovereign due to lower borrowing costs. Since seniority does not affect the payoff from permanent autarchy, the higher value of market access translates into a higher surplus for the sovereign for any given level of G. Nash bargaining leads to a transfer of a portion of this gain in surplus to creditors. Figure 2 displays the renegotiated debt level following default for the two market arrangements.

Second, Table 4 also indicates a substantial decline in the volatility of spreads when seniority is enforced. However, when scaled by their mean value, spreads are somewhat more volatile when seniority is enforced: The coefficient of variation when seniority is enforced is 0.51 as opposed to 0.47 when it is not. This seems intuitive given that, in the region where spreads are sensitive to the level of debt, the sensitivity is greater when seniority is enforced.

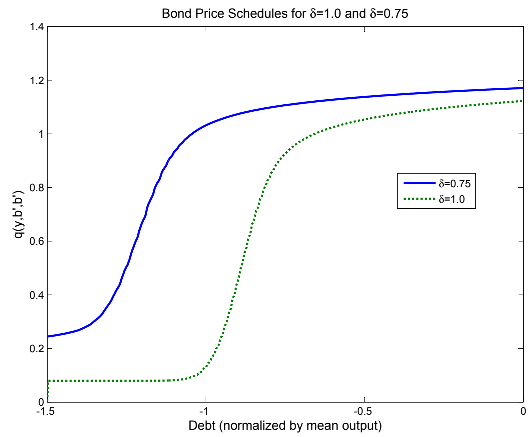
We close this section with a comparison of our findings with the results reported in Hatchondo, Martinez & Sosa-Padilla (2011). As mentioned in our introduction, these authors quantify the effects of debt dilution for spreads and the frequency of default by considering an idealized market arrangement in which the sovereign compensates existing creditors for any loss in market value of outstanding debt that results from additional borrowing. They find that such an arrangement raises welfare by about 0.14 percent in consumption equivalents (see their Figure 2, p. 18). This estimate is an order of magnitude smaller than the welfare gain reported in our Table 4. We suspect that this difference in findings is due, in part, to the fact that these authors target a much lower debt-to-output ratio than we do (0.28 versus 1.0). Since the sovereign carries more debt in our model, the wealth effect from a drop in interest rates is much larger in our model than in theirs. Also, since the magnitude of the default costs influences the average debt-to-output ratio (higher default costs mean higher debt capacity), a default is more costly in our model. This implies that the drop in default frequency has a greater positive welfare impact in our model.

7.3 Sensitivity

In this section we investigate two issues that might bear importantly on the size of the welfare gain reported in Table 4. One issue is the sensitivity of the estimate of the welfare gain to the size of the upper bound on one-period-ahead default probability, namely δ. Would the estimate of the welfare gain from enforcing seniority decline if underwriting standards are substantially tightened (δ is lowered)? The second issue is whether prior debt substantially reduces that welfare gain from enforcing seniority. The reason for considering this possibility is that, practically speaking, seniority can be introduced only if existing creditors are made senior to new creditors.[26] But making all existing debt senior to any new debt that might be issued in the future

[26]Sovereign-debt contracts typically include a *pari passu* clause, which stipulates that existing creditors cannot be treated any worse than new creditors.

Figure 3: Impact of Underwriting Standards on Bond Prices



will result in a wealth transfer from the sovereign to existing creditors, potentially negating the welfare gain reported above.

7.3.1 The Role of "Underwriting Standards"

We begin by discussing what happens to the bond price function when the underwriting standards are eliminated, or, equivalently, the value of δ is set to 1.

The bond price function with $\delta = 1$ is plotted in Figure 3, along with the bond price function for the baseline calibration ($\delta = 0.75$). Observe that the price function without the standard is uniformly lower. To understand why, recall that, without the underwriting constraint, the sovereign prefers to dilute existing debt maximally just prior to default. This behavior implies several things. First, since creditors have rational expectations, they take into account that, in the event of default, they will receive very little by way of repayment: b' will explode prior to default, and the recovery rate, namely $G(y')/b'$, will be almost zero for all values of y'. Second, the fact that the sovereign is able to essentially promise all repayment in default to new creditors (the ones who purchase the enormous quantities of bonds issued just prior to default) means that the sovereign gets a consumption boost in

Table 5: Impact of Underwriting Standards

Statistic	$\delta = 0.75$	$\delta = 1.0$
Welfare in Consumption Equivalent ($b = 0$)	1.0282	1.0174
Average Annualized Spread	0.0809	∞
Std. Dev of Annualized Spreads	0.0380	–
Annualized Default Probability	0.0853	0.0877
Recovery Rate	0.30	0
Average Debt-to-Output Ratio	1.00	∞

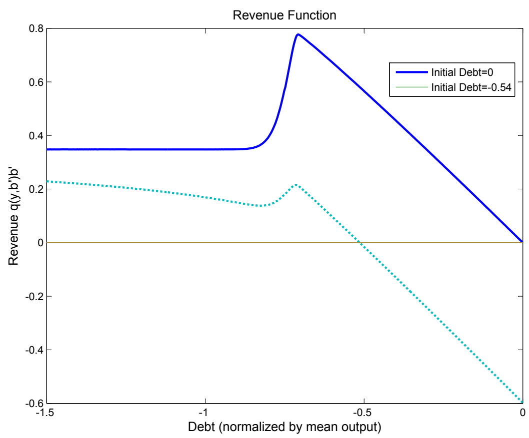
the period preceding default. In effect, the expropriation of existing creditors right before default lowers the cost of default to the sovereign and thereby elevates the frequency of default. Indeed, as reported in Table 5, the frequency of default is higher when the underwriting constraint is removed.

The lack of any recovery in the event of default and the higher frequency of default explain why the $q(y, b')$ schedule is uniformly lower when underwriting standards are eliminated. One might think that this fact will be reflected in the average spread statistic for this economy, but this statistic is essentially unbounded in this case: Because of maximal dilution, b' hits the lower bound on the B grid right before default, and the equilibrium $q(y, b')$ is essentially zero (and the spread infinite) in that period. For the same reason, the average debt-to-output ratio is also essentially unbounded. Regardless, the fact that borrowing costs are uniformly higher in this economy is reflected in the lower value of the constant consumption equivalent. It drops from 1.0282 (for $\delta = 0.75$) to 1.0174. The welfare gain from enforcing seniority is correspondingly higher: The gain is now almost 2 percent of consumption in perpetuity.

Next, we examine what happens when δ is lowered to 0.50. It turns out that there is no change in model statistics relative to the baseline case. The model behaves in exactly the same way as when $\delta = 0.75$.

To understand this (initially) surprising result, it helps to look at how the revenue curve from new bond sales, namely $q(y, b')[(1-\lambda)b - b']$, behaves with respect to prior debt b. The solid line in Figure 4 plots the revenue curve for the case where there is no

Figure 4: Impact of Prior Debt on the Revenue Function



prior debt. The curve has the familiar hump shape: Revenue is initially increasing in bond sales until the price of each bond sold drops enough so that total revenue begins to decline. Because of repayment, the revenue curve flattens out at some positive value for high levels of debt: At these debt levels, the probability of default next period is 1 and, in this case, the value of debt is simply $E_{y'|y}q(y', G(y'))G(y')/(1+r_f)$ and independent of b'. Figure 4 also plots the revenue curve when there is prior debt. Observe that the curve now has a segment that is rising in debt for large enough levels of debt. This portion of the revenue curve results from the logic explained earlier: When default is certain, the sovereign can expropriate existing creditors by increasing b'. Indeed, as the level of prior debt increases, maximum dilution (and default in the next period) generates more current revenue than maximum revenue associated with the top of the revenue hill.

We can now understand how δ affects the sovereign's behavior. Basically, a value of δ less than 1 means that the sovereign is limited in how far out (to the left) it can go in terms of debt issuance. If δ is large enough, the sovereign may find it in its interest to engage in maximum dilution and push bond sales all to the point where δ is binding. If δ is lowered enough so that the revenue obtained from maximum dilution is just less than the revenue associated with the top of the revenue hill, the incentive to engage in maximal dilution goes away entirely. Importantly, lowering δ further will not change the sovereign's behavior. Thus, there is a cutoff level of δ below which the incentive to engage in maximal dilution is absent for all debt levels reached in equilibrium and above which it is present and active for at least one (y, b) pair reached in equilibrium. Our calibration of δ is right near this cutoff value. That is why lowering the value of δ makes no difference to model statistics. In effect, we have dropped δ below 1 by the minimum amount necessary to (barely) eliminate the incentive to engage in maximal dilution right before default.[27]

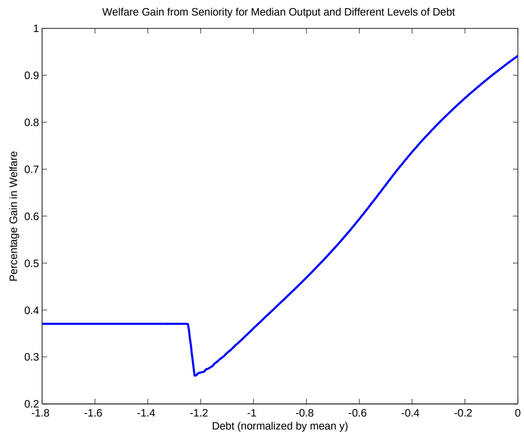
7.3.2 Does Prior Debt Negate the Welfare Benefits of Seniority?

Consider the sovereign (Argentina) contemplating introducing seniority at a time when its endowment is at its median level y^m and it has some prior debt b. Assume that if seniority is introduced, each outstanding (unit) bond is randomly assigned a rank between 0 and b and that all new issuances end up ranked below the lowest-ranked prior debt.[28] Figure 5 shows the welfare gains to Argentina from switching to a seniority regime for different levels of prior debt. As the level of prior debt increases, the welfare gain from imposing seniority drops. It drops because imposing seniority makes existing debt more valuable, which is to say that the sovereign promises to pay out more on these bonds in the future. Since the sovereign's real resources haven't changed, this promise is, in effect, a transfer of wealth from the sovereign to its existing creditors.

[27] Although not reported in Table 5, this finding holds all the way to $\delta = 0.33$. If δ is lowered further, there is a decrease in spreads, in the debt-to-output ratio, in the default frequency, and in the average recovery rate.

[28] An alternative would be to treat all existing bonds as equally senior. This would lead to a common price for existing bonds when seniority is imposed. However, if under either arrangement the sovereign never buys back its existing debt, its behavior going forward will be the same regardless of which alternative is followed. This is because what matters then for the price of new bonds is only that they are junior to existing bonds. Accordingly, the total value of existing debt will be the same regardless of which arrangement is followed. Since lenders are risk-neutral, they should be indifferent between which arrangement is followed. We follow the first arrangement because it is easier to implement numerically.

Figure 5: Impact of Prior Debt on the Welfare Gain from Enforcing Seniority



Two remarks are worth making. First, the sovereign does not have to bear this loss in wealth fully. Since existing creditors gain from being made senior to all future creditors, they may be willing to share this gain with the sovereign. The sharing could take the form of a voluntary debt exchange wherein old debt is exchanged for new debt at less than par (in other words, a holder of a bond with a face value of 1 would receive a new bond with a face value of, say, 0.95). Second, even if we assume that creditors do not share the gain with the sovereign, Figure 5 shows that imposing seniority continues to be welfare improving for fairly large levels of prior debt. In fact, for the median output level, there is no level of prior debt that leads to a loss in welfare from imposing seniority.[29] Thus, quantitatively speaking, the presence of prior debt does not appear to be a major hurdle to incorporating a seniority clause in sovereign debt contracts.

[29]Initially, the drop in welfare is concave with respect to the level of prior debt, but then it becomes almost linear. The linearly dropping portion of the graphs begins at the point where, in the absence of seniority, the sovereign wants to default. The point where the graphs jumps up is where the sovereign defaults even when seniority is imposed.

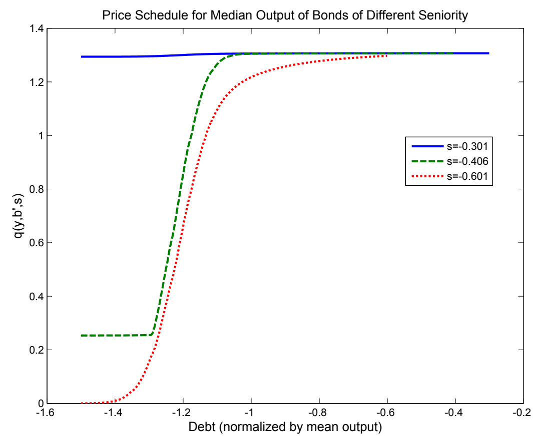
8 When Is Seniority a Solution to the Debt Dilution Problem?

In this section our objective is to better understand why enforcing seniority does not completely eliminate dilution in our quantitative model and to describe an environment for which seniority fully solves the debt dilution problem. By "solving the debt dilution problem" we mean that the equilibrium price function q^* has the property that, given any y and s, $q^*(y, s, s) = q^*(y, b', s)$ for all $b' < s$. In other words, the price of a unit bond with given seniority s is unaffected by additional borrowing by the sovereign.

To start off, we want to make explicit that enforcing seniority does not eliminate dilution in our quantitative model. This point is clearly shown in Figure 6, which plots the $q(y, b', s)$, $b' \leq s$, as a function of b' (given y at its median value) for three different values of s. The solid line shows the schedule for $s = -0.3$. This schedule is defined for all $b' \leq -0.3$ and is essentially risk-free up until the debt level of about -1 and then experiences a small decline in value as b' falls further. The dashed line shows the schedule for $s = -0.4$, which is defined for all $b' \leq -0.4$. For this bond, the price concides with the $s = -0.3$ bond up until $b' \approx -1.0$ and then there is a substantial drop-off in value as b' falls further. Finally, the dotted line shows the price schedule for $s = -0.6$. This bond is junior enough that its value is adversely affected by borrowing beyond $b' = -0.6$.

We now turn to describing an environment where seniority completely overcomes debt dilution. The main difference between this environment and the one described in the previous section is what happens in default. First, there are no output losses from default, i.e., $\phi(y)$ is now identically zero. Second, the sovereign is required to service the restructured debt $G(y)$ in the period of default, i.e., the sovereign must pay out $[\lambda + (1 - \lambda)z]G(y)$ to its creditors in the period of default. Third, the sovereign can access capital markets immediately following agreement on the new

Figure 6: Impact of New Borrowing on Existing Debt of Varying Seniority



level of debt, i.e., the debt level at which the sovereign exits the default period can be different from $G(y)$. And, fourth, debt buybacks are not permitted.

For the arguments to follow, the value of $G(y)$ is not important as long as it is strictly positive for all y (i.e., there is some repayment in the event of default). So we will take $G(y) > 0$ as parametrically given. Adhering to the new default rules, the payoff from default is now

$$\begin{aligned} & X(y, G(y)) = \max_{b' \in B} u(c) + \beta E_{y'|y} \max\{V(y', b'), X(y', G(y'))\} \\ & \text{s.t.} \\ & c = y + [\lambda + (1-\lambda)z]\, G(y) + R(y, G(y), b') \\ & b' \leq (1-\lambda)G(y). \end{aligned} \tag{13}$$

Observe that the sovereign must service the level of debt agreed to in the renegotiation during the period of default. Following the renegotiation, it can choose a new debt level subject to the no-buyback constraint.

The payoff from repayment is

$$
\begin{aligned}
&V(y,b) = \max_{b' \in B} u\left(c\right) + \beta E_{(y'|y)} \max\{V(y',b'), X(y',G(y'))\} \\
&\text{s.t.} \\
&c = y + \left[\lambda + (1-\lambda)z\right] b + R(y,b,b') \\
&b' \leq (1-\lambda)b.
\end{aligned} \tag{14}
$$

A comparison of these two value functions immediately shows that the payoff from default, $X(y, G(y))$, is simply $V(y, G(y))$. Since $V(y, b)$ is strictly increasing in b (given $b^0 > b^1$, all choices that are feasible for b^0 are also feasible for b^1 and afford strictly greater consumption), it follows that the sovereign will choose to default whenever $b < G(y)$. Taking account of this, we can rewrite the value from repayment entirely in terms of the $V(y, b)$ function:

$$
\begin{aligned}
&V(y,b) = \max_{b' \in B} u\left(c\right) + \beta E_{(y'|y)} V(y', \max\{b', G(y')\}) \\
&\text{s.t.} \\
&c = y + \left[\lambda + (1-\lambda)z\right] b + R(y,b,b') \\
&b' \leq (1-\lambda)b.
\end{aligned} \tag{15}
$$

This decision problem delivers the debt decision rule in the event of repayment $a(y, b)$. The default decision rule is implicit in the statement of the problem: $d(y, b)$ =1 if and only if $b < G(y)$.

The pricing equation reflecting the new default arrangement is now

$$
\begin{aligned}
q(y,b',s) = E_{y'|y}&\left[[1-d(y',b')]\frac{\lambda + [1-\lambda][z + q(y', a(y',b'), [1-\lambda]s)]}{1+r_f}\right] \\
&+ E_{y'|y}\left[d(y',b')\mathbb{1}_{\{s \geq G(y')\}}\frac{\lambda + [1-\lambda][z + q(y', a(y',G(y')), [1-\lambda]s)]}{1+r_f}\right]
\end{aligned} \tag{16}
$$

In the event there is no default, the evolution of seniority of a non-maturing bond is simply $(1-\lambda)s$ since buybacks are not permitted. In the event of default, the bond either pays nothing (if $s < G(y')$) or it pays 1 with probability λ and $z + q(y', a(y', G(y')), [1-\lambda]s)$ with probability $(1-\lambda)$.

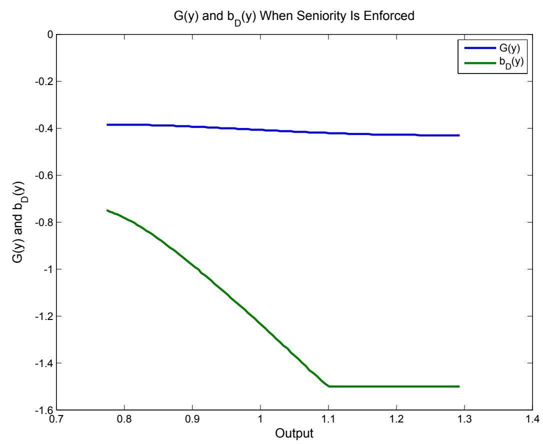
We can now state our main result for this environment.

Proposition 2 *Given any $G(y) > 0$, the equilibrium price function $q^*(y, b', s)$ is dilution-free.*

To gain intuition for this result, consider the following situation. Suppose that the sovereign begins the period with debt b_0 and issues more debt, say, to a level $b_1 < b_0$. Suppose that this additional borrowing increases the probability of default next period, i.e., there exists $\hat{y} \in Y$ such that there is no default if $\hat{y}$ is realized and the debt is b_0, but there is default if the debt is b_1. We want to see how the value of an *existing* unit bond with some particular ranking $\tilde{s} \geq b_0$ is affected by the increase in default probability induced by this additional borrowing. Consider what happens to the payoff of a $\tilde{s}$ bond if $\hat{y}$ is realized next period. The debt is immediately written down to $G(\hat{y})$. Since $b_0 > G(\hat{y})$ — there is no default for $\hat{y}$ when the debt is b_0 — the $\tilde{s}$ bond is part of the settlement. Thus, the elevated default probability does not directly affect the payout on any bond with $\tilde{s} \geq b_0$.

What is special about this situation is that $G(\hat{y})$ is playing a dual role: It is the debt level at which default is triggered for $\hat{y}$ and, following default, it is also the debt level agreed to in the renegotiation. This coincidence is the result of there being no resource loss from default. If default were costly ($\phi(\hat{y}) > 0$), the level of debt at which default is triggered — call it $b_D(\hat{y})$ — will be higher than $G(\hat{y})$: The sovereign will not default for debt levels between $b_D(\hat{y})$ and $G(\hat{y})$ because the gain from reducing its debt level is more than offset by the output lost due to default. This wedge between $b_D(\hat{y})$ and $G(\hat{y})$ opens up the possibility that an increase in indebtedness can directly affect the payout on an existing bond. To see this, consider $b_1 < b_D(\hat{y}) < b_0 < G(\hat{y})$. Once again, there is no default on b_0 if $\hat{y}$ is realized, but

Figure 7: The Wedge Between the Default Trigger and the Renegotiated Debt



there is default if the level of debt is b_1. But, now, if $\hat{y}$ is realized, then any bond with $\tilde{s} \in [b_0, G(\hat{y}))$ is out of the settlement. Thus, all these bonds will lose value due to the increase in debt from b_0 to b_1.

Indeed, it is the gap between $b_D(y)$ and $G(y)$ that accounts for why seniority cannot completely eliminate dilution in our quantitative exercise. Figure 7 plots the value of the $b_D(y)$ and $G(y)$ for different levels of y for our baseline economy. As is evident, $b_D(y)$ is significantly less than $G(y)$. This wedge is the result of the various costs of default assumed in our baseline calibration. Lowering default costs would narrow this wedge and make seniority more effective in preventing dilution.

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A Appendix

In this appendix we give the proofs of Propositions 1 and 2. In preparation, we prove one Lemma that will be used in the proofs. The Lemma establishes that, *given* a pair of decision rules $a(y, b)$ and $d(y, b)$, the pricing equations (7) and (16) define contraction maps on the space of $q(y, b', s)$ functions.

Definitions. Let W be the set of all $(y, b', s) \in Y \times B \times [b_I, 0]$ such that $b' \leq s$ and let w denote an element of W. Let Q denote the set of all bounded and nonnegative functions $q(w) : W \to \mathbb{R}^+$. Let $\rho(q, \tilde{q}) = \sup_w |q(w) - \tilde{q}(w)|$ be the metric on Q. Then (Q, ρ) is a complete metric space (for a proof, see, for instance, Harris (1987), Lemma 2.1, p. 22). Let $(Hq)(y, b', s) : Q \to Q$ be the operator defined by the r.h.s. of (7) and let $(Tq)(y, b', s)$ be the operator defined by the r.h.s of (16).

Lemma 1 *Given decision rules, H and T are contraction maps with modulus* $1/(1+r_f)$.

Proof. We verify that H and T satisfy Blackwell's suffcient conditions for a contraction.

(i) Monotonicity: Consider $q^1(y, b', s) \leq q^0(y, b', s)$, both members of Q. It is clear from inspection that $(Hq^1) \leq (Hq^0)$ and $(Tq^1) \leq (Tq^0)$.

(ii) Shrinkage: Let $\kappa > 0$. Then

$$(H(q+\kappa)) = (Hq) + \frac{\kappa}{1+r_f}\left[(1-\lambda)]E_{y'|y}[1-d(y',b')] + E_{y'|y}d(y',b')\mathbb{1}_{s\geq G(y')}\right]$$

$$(T(q+\kappa)) = (Tq) + \frac{\kappa}{1+r_f}(1-\lambda)\left[E_{y'|y}[1-d(y',b')] + E_{y'|y}d(y',b')\mathbb{1}_{s\geq G(y')}\right]$$

In either case, the term multiplying $\kappa/(1+r_f)$ is strictly less than 1. Hence, $H(q+\kappa) < H(q) + \kappa/(1+r_f)$ and $T(q+\kappa) < H(q) + \kappa/(1+r_f)$. ■

We are now ready to the give proofs of Propositions 1 and 2.

Proof of Proposition 1. Let $q^*(y, b', s)$ be the equilibrium pricing function. Let

$d^*(y,b)$ and $a^*(y,b)$ be the equilibrium default and asset decision rules, respectively. Then, this 3-tuple together satisfies equation (7). Let H^* denote the operator defined by the r.h.s. of (7), where the asterisk indicates that the operator is being defined for the equilibrium decision rules.

It is clear from inspection that H^* preserves monotonicity with respect to s: If $q(y,b',s)$ is increasing in $s \geq b'$, then $(H^*q)(y,b',s)$ is increasing in $s \geq b'$. Now, note that the set $Q'(W) = \{q \in Q : q \text{ is increasing in } s\}$ is a closed subset of Q. It follows from Lemma 1 above and the Theorem 3.1 and Corollary 1 to Theorem 3.1 of Stokey & Lucas (1989)(pp. 50-52) that there is a *unique* $\bar{q} \in Q'$ such that $(H^*\bar{q})(w) = \bar{q}(w)$. Since $q^*(y,b',s)$ satisfies equation (7), $q^*(y,b',s)$ must be $\bar{q}(w)$. Therefore $q^*(y,b',s)$ must be increasing in s. ■

Proof of Proposition 2. We have to show that for each y and s

$$q(y,s,s) = q(y,b',s) \text{ for all } b' < s. \tag{17}$$

For convenience, we repeat the pricing equation for this case here:

$$\begin{aligned} &q(y,b',s) = \\ &E_{y'|y}\Bigg[[1-d^*(y',b')]\frac{\lambda+[1-\lambda][z+q(y',a^*(y',b'),[1-\lambda]s)]}{1+r_f} + \\ &\qquad d^*(y',b')1_{\{s\geq b_D(y')\}}\frac{\lambda+[1-\lambda][z+q(y',a^*(y',G(y')),[1-\lambda]s)]}{1+r_f}\Bigg] \end{aligned} \tag{18}$$

Denote the r.h.s. of the above equation as the operator T^*. We will show that if q satisfies (17), then $(T^*q)(w)$ satisfies (17) also, i.e., $(T^*q)(y,b',s) = (T^*q)(y,s,s)$ for all $b' < s$. To this end, fix y and s and consider $b' < s$.

Let y' be such that $d^*(y',s) = d^*(y',b') = 0$ (i.e., y' is a state in which there is repayment for both debt levels). By the no-buyback restriction, $a^*(y',b') \leq (1-\lambda)b' < (1-\lambda)s$ and $a^*(y',s) \leq (1-\lambda)s$. By (17), $q(y',a^*(y',b'),[1-\lambda]s) = q(y',a^*(y',s),[1-\lambda]s)$.

Next, suppose y' is such that $d^*(y', s) = 0$ but $d^*(y', b') = 1$. Then, the payoff when debt is s will be $\lambda + [1 - \lambda][z + q(y', a^*(y', s), [1 - \lambda]s)]$ and the payoff when debt is b' will be $\lambda + [1 - \lambda][z + q(y', a^*(y', G(y')), [1 - \lambda]s)]$. But $G(y') \leq s$ (otherwise, $d^*(y', s)$ would be 1, not 0). By the no-buyback restriction, $a(y', s) \leq (1 - \lambda)s$ and $a(y', G(y') \leq (1 - \lambda G(y)) \leq (1 - \lambda)s$. By (17), we have that $q(y', a^*(y', G(y')), [1 - \lambda]s) = q(y', a^*(y', s), [1 - \lambda]s)$.

Next, suppose that y' is such that $d^*(y', s) = d^*(y', b') = 1$. Then, the payoff on the debt (of seniority s) is $\lambda + [1 - \lambda][z + q(y', a^*(y', G(y')), [1 - \lambda]s)]$ regardless of whether the debt is s or b'.

Since $d^*(y, b)$ is decreasing in b, these are the only three cases we need to examine.

Putting the results together, we conclude that $(T^*q)(y', b', s) = (T^*q)(y', s, s)$ for all $b' < s$. To complete the proof, observe that the set $Q' = \{q \in Q : q \text{ satisfies (17) }\}$ is a closed subset of Q. It follows (from Lemma 1 above and the Theorem 3.1 and Corollary 1 to Theorem 3.1 of Stokey & Lucas (1989)) that there is a *unique* $\bar{q}$ such that $(T^*\bar{q}) = \bar{q} \in Q'(w)$. Since $q^*(y, b', s)$ satisfies (17), $q^*(y, b', s)$ must be $\bar{q}(w)$. Therefore, $q^*(y, b', s)$ must satisfy (17) as well. ∎