Firm Dynamics, Job Turnover, and Wage Distributions in an Open Economy *

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Abstract

This paper explores the effects of tariffs, trade costs, and firing costs on firm dynamics and labor markets outcomes. The analysis is based on a general equilibrium model with labor market search frictions, wage bargaining, firing costs, firm-specific productivity shocks, and endogenous entry/exit decisions. Firing costs reduce firms’ profits and discourage them from quickly adjusting their employment levels in response to idiosyncratic shocks. Tariffs and other trade costs reduce rents for efficient firms and increase rents for inefficient firms, as in Melitz (2003). These well-known effects interact with idiosyncratic productivity shocks and with scale economies in hiring costs to determine the equilibrium size distribution of firms, entry/exit rates, job turnover rates, rate of informality, and cross-firm wage distributions.

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1 Introduction

Wage inequality, job insecurity, high unemployment, and large informal sectors are long-standing concerns in Latin America. During the 1990s, following a wave of trade liberalization episodes and labor market reforms, these problems grew worse for many countries in the region.\footnote{Inter-American Development Bank (2004) summarizes the deterioration in Latin American labor market conditions and notes that “Between the mid-1980s and the beginning of the 1990s, countries in Latin America began trade liberalization programs, with reductions of at least 15 percentage points in the average tariff rate, which fell from an average of 48.9% in the pre-reform years to 10.7% in 1999.” (p. 137). Heckman and Pages (2004) survey labor market regulations in Latin America, observing that “the new openness to international trade increased the demand for labor market flexibility.” They point to Argentina, Colombia, Ecuador, Nicaragua and Peru as examples of countries that fit this pattern. Haltiwanger et al (2004) document the association between job turnover and openness in Latin America. Goldberg and Pavcnik (2007) survey the evidence linking openness to wage inequality and informality in Latin America and other developing regions.} But the extent to which these countries’ trade and labor policy reforms contributed to deteriorating labor market conditions remains an open question. Many other other forces were also in play, including large macro shocks, skill-biased technological progress, privatization, and changes in global markets (Inter-American Development Bank, 2004).

We propose and calibrate a new model that characterizes the long run effects of commercial policy and labor market reforms in the absence of these confounding factors. Several of the mechanisms at work in our formulation are well-known. In particular, openness moves workers toward firms with low marginal costs, and reductions in firing costs make firms’ sizes more responsive to their idiosyncratic shocks. However, we depart from earlier models in several key respects. First, shifts in the size distribution affect job turnover rates because growth rates and exit rates are dramatically higher at small firms.\footnote{For example, using the U.S. Census Bureau Business Dynamics Statistics and Longitudinal Business Database, establishments, Haltiwanger et al (2010, figure 7) calculate that job creation rates and job destruction rates range from 20 to 35% for producers with less than 10 workers, fall to 15-25% among establishments with 10-50 workers, and average around 15% among producers past the 50 worker threshold. Neumark, et al (2011) report qualitatively similar patterns based on the U.S. National Establishment Time Series data base.} Second, openness induces some firms to switch from exclusively serving the domestic market to serving multiple markets, and in doing so increases their incentives to hire or fire workers in response to idiosyncratic productivity shocks. Finally, these effects both interact with labor market reforms that reduce the costs of shedding workers.

We quantify these effects by fitting our model to plant-level panel data from Colombia — a country that liberalized trade, deregulated labor markets, and nearly tripled the share of exports in its total manufacturing sales between the late 1980s and
the early 2000s. Our calibrated model closely replicates basic features of Colombian micro data in the decade preceding reforms, including the size distribution of firms, the rates of turnover among firms of different sizes, producer entry and exit rates, exporting patterns, and the serial correlation in firm-level employment levels.

Experiments based on the calibrated model yield several basic findings. First, Colombia’s tariff reductions modestly increased wage dispersion with little effect on job security. Second, reductions in global trade frictions - driven by the opening of external markets and reduced transport/communication costs - compounded the increase in wage inequality and tended to increase job turnover. Third, the reductions in firing costs that Colombia implemented reinforced the effects of globalization on job turnover and contributed to rising rates of informality. Finally, however, despite the greater turnover and wage dispersion, the combined effects of reforms and reductions in trade costs was to increase workers’ welfare, particularly those employed at large, efficient firms.

Our model draws on at least three literatures. First, it shares some basic features with multi-worker firm models in the labor-search literature. In particular, it can be viewed as an extension of Bertola and Caballero (1994), Bertola and Garibaldi (2001) and Koeniger and Prat (2007) to include fully articulated product markets, international trade, arbitrary (stationary) Markov processes for productivity shocks and endogenous firm entry and exit.\(^3\) Second, it shares some characteristics with a number of recent trade models that describe the effects of openness on labor markets (Helpman and Itskhoek (2010); Helpman et al (2010); Egger and Kreickemeier (2007); Amiti and Davis (2008); Davis and Harrigan (2008)); Felbermayr et al (2008).\(^4\) Most notably it embodies Melitz’s (2003) basic insight that openness compounds the advantages enjoyed by relatively efficient firms, and it translates these changes in the

\(^3\)Other recent papers that study firm dynamics and labor market frictions in a closed economy context include Cooper et al (2007), Lentz and Mortensen (2010), and Hobijn and Sahin (2010). Utar (2008) studies firm dynamics and labor market frictions in an import-competing industry that takes the wage rate as given.

\(^4\)Several less-related linkages between openness and labor market outcomes have been modeled in the recent trade literature. One strand of this literature emphasizes the changes in skill-premia and/or unemployment rates that result from trade-induced changes in the relative demand for different types of labor (e.g., Albrecht and Vroman (2002), Yeaple (2005), Davidson et al (2008)). Another characterizes the adjustments in wages, unemployment and labor turnover patterns that derive from trade-induced changes in sectoral output prices (e.g., Kambourov (2009), Artuç, Chaudhuri and McLaren (2010)). Third, some studies have focussed on cross-country differences in the flexibility of labor markets as a source of comparative advantage (Davidson et al (1999), Cunat and Melitz (2007), Helpman and Itskhoek (2010)). Finally, Holmes and Stevens (2010) abandon the standard Melitz (2003) mechanism in favor of the assumption that large firms are relatively hurt by openness because they produce standardized products that compete head to head with imports, while small firms produce customized “boutique” goods that foreign suppliers cannot easily replicate.
relative profitability of different producers into associated changes in the wage distribution. Finally, our formulation draws on Hopenhayn’s (1992) characterization of firm dynamics, and in that sense it is related to many previous models that generate size-dependent volatility, including Jovanovic (1982), Ericson and Pakes (1995), Klette and Kortum (2004), Luttmer (2007), and Rossi-Hansberg and Wright (2007).

While we do not pretend to capture all of the channels through which openness and firing costs can affect labor market outcomes, our focus on firm-level entry, exit and idiosyncratic productivity shocks is supported by existing empirical evidence on the sources of job turnover and wage heterogeneity. Studies of job creation and job destruction invariably find that most reallocation is due to idiosyncratic (rather than industry-wide) adjustments (Davis et al (1998), Roberts (1996), Inter-American Development Bank (2004)). “This is true even in Latin America’s highly volatile macro environment” where producer entry and exit alone account for 30-40% of job creation and destruction (Inter-American Development Bank (2004), chapter 2). Further, as Goldberg and Pavcnik (2007) note, if openness has had a significant effect on job flows, it has mainly been through intra-sectoral effects: “Most studies of trade liberalization in developing countries find little evidence in support of [trade-induced labor] reallocation across sectors.” Finally, while cross-worker differences in wages are obviously partly due to differences in worker characteristics, much is attributable to labor market frictions and firm heterogeneity.$^5$

2 The Model

2.1 Preferences

We consider an economy populated by a fixed supply of homogeneous, infinitely-lived worker-consumers who purchase two types of output: homogeneous services and differentiated industrial goods. These agents derive no disutility from work, and their consumption preferences are given by

$$U = \sum_{t=1}^{\infty} \frac{s_t^{1-\gamma} q_t^\gamma}{(1+r)^t}, \quad \gamma \in (0, 1),$$

$^5$Studying data from France and the United States, Abowd et al. (1999) and Abowd et al. (2002) show that roughly half of the cross-worker variation in compensation in French workers is due to employer effects. Menezes-Filho et al. (2008) use matched employer-employee data from Brazil and find that establishment fixed effects constitute a smaller share of overall wage variation in Brazil compared to France and the United States.
where \(1 + r\) is the discount rate, \(s_t\) is consumption of services, and \(q_t\) is an index of industrial good consumption. Preferences over individual industrial goods are of Dixit-Stiglitz type

\[
q_t = \left( \int_{0}^{N_t} q_t(n) \frac{2^{1/\sigma}}{2^{1/\sigma} - 1} \, dn \right)^{\frac{\sigma}{\sigma - 1}}.
\]

(1)

Here \(N_t\) measures the mass of differentiated good varieties at time \(t\), \(q_t(n)\) is consumption of good \(n\) at time \(t\) and \(\sigma > 1\) is the elasticity of substitution between varieties.

Services are non-traded, but \(N_F\) of the \(N_t\) differentiated goods are produced abroad. Suppressing time subscripts, and letting \(p_F(n)\) be the foreign-currency denominated FOB price of imported variety \(n\), the exact price index for imported goods is

\[
P_F = \tau_m \tau_c k \left( \int_{0}^{N_F} p_F(n)^{1-\sigma} \, dn \right)^{1/(1-\sigma)},
\]

where \(k\) is the exchange rate, \((\tau_c - 1)\) is the iceberg transport cost per unit shipped and \((\tau_m - 1)\) is the ad valorem tariff rate on imports. Similarly, letting \(p_H(n)\) be the price of domestic variety \(n\), the exact price index for domestic goods is

\[
P_H = \left( \int_{N_F}^{N} p_H(n)^{1-\sigma} \, dn \right)^{1/(1-\sigma)}.
\]

Several normalizations simplify notation. First, since the set of available foreign varieties and their FOB prices are exogenous to our model, we normalize

\[
\left( \int_{0}^{N_F} p_F(n)^{1-\sigma} \, dn \right)^{1/(1-\sigma)}
\]

to unity by choice of foreign currency units. This allows us to write the exact price index for the composite industrial good \(q\) as

\[
P = [P_{H}^{1-\sigma} + (\tau_m \tau_c k)^{1-\sigma}]^{1/(1-\sigma)}.
\]

(2)

Second, without loss of generality we choose the price of services to be our numeraire.

Disallowing savings, utility maximization implies that units of domestic variety \(n\) consumed by worker \(i\) with income \(I_i\) is

\[
q_{H_i}(n) = \frac{\gamma I_i}{P} \left( \frac{p_H(n)}{P} \right)^{-\sigma},
\]
and demand for imported variety $n'$ by the same type of worker is

$$q_{F_i}(n') = \frac{\gamma I_i}{P} \left( \frac{\tau_m \tau_c kp_F(n')}{P} \right)^{-\sigma}.$$  

Aggregating over worker-consumers, these expressions in turn imply that total domestic demand for domestic variety $n$ is

$$q_H(n) = \frac{1}{0} q_{Hi}(n)di = D_H p_H(n)^{-\sigma},$$  

where $D_H = \gamma P^{\sigma-1} \int_0^1 I_i di$ and the mass of domestic worker-consumers is normalized to measure one. Similarly, total domestic demand for imported good $n'$ is

$$q_F(n') = \frac{1}{0} q_{Hi}(n')di = D_H [\tau_m \tau_c kp_F(n')]^{-\sigma}.$$  

### 2.2 Production Technologies

Services are supplied by service-sector firms and, less efficiently, by unemployed workers engaged in home production. Regardless of their source, services are produced with labor alone, homogeneous across suppliers, and sold in competitive product markets. Firms that supply services use a common constant returns technology, and face no hiring or firing costs. So with an appropriate choice of output units we may write their combined supply of services as

$$S = L_S,$$

where $L_S$ is labor employed in the service sector. Unemployed workers who home-produce service goods each generate $b < 1$ units of output.

Industrial goods cannot be home-produced. They must be supplied by industrial-sector firms, which pay a sunk start-up cost to initiate production of a single variety of output. Each firm produces its output using labor alone and competes in the monopolistically competitive product market. Unlike service-sector firms, suppliers of industrial goods are subject to ongoing idiosyncratic productivity shocks, and they must create costly vacancies in order to attract new workers. (As in Melitz (2003), productivity variation can equally well be thought of as variation in product appeal.) In the industrial sector, output of producers with productivity level $z$ is given by

$$q(z,l) = z l^\alpha,$$
where $l$ is the labor input and $\alpha > 0$. Productivity is firm-specific, independent across firms, and serially correlated. Its evolution is characterized by the transition density $h(z'|z)$, which is common to all firms. Productivity shocks together with firms’ employment policies and entry/exit policies determine the steady state distribution of firms across $(z,l)$, which we denote by $\psi(z,l)$.

Producer dynamics in the industrial sector resemble those in Hopenhayn (1992) and Hopenhayn and Rogerson (1993) in that firms react to their productivity shocks by optimally hiring, firing or exiting. Also, new firms enter whenever their expected future profit stream exceeds the entry costs they face. However, unlike these authors, we assume hiring in the industrial sector is subject to search frictions captured by a standard matching function. Labor market frictions generate rents that are bargained over between worker and firms, and firms end up paying different wages depending on their current productivity and labor force, as well as whether they are hiring or firing workers. Further, workers maximize the present value of their expected welfare by making forward-looking decisions concerning which sector to work in and what job offers to accept. We now describe the functioning of labor markets in more detail.

### 2.3 Labor Markets and the Matching Technology

The service sector labor market is frictionless so, given that the price of services is unity, the service sector wage is $w_s = 1$. Search frictions make things more complicated in the industrial sector. Each period the number of new matches between unemployed workers and vacancy posting firms is given by

$$M(V, L_u) = \frac{V L_u}{(V^\theta + L_u^\theta)^{1/\theta}},$$

where $L_u$ is the measure of unemployed workers searching in industrial sector and $V$ is the measure of vacancies in industry.\(^6\) Consequently, industrial firms fill each vacancy they post with probability

$$\phi(V, L_u) = \frac{M(V, L_u)}{V} = \frac{L_u}{(V^\theta + L_u^\theta)^{1/\theta}},$$

\(^6\)The functional form of the matching function follows den Haan et al. (2000). It is constant returns to scale, and increasing in both arguments. In contrast to the standard Cobb-Douglas form, it has no scale parameter and the implied matching rates are bounded between zero and one.
while unemployed workers searching for industrial jobs find matches with probability

\[ \tilde{\phi}(V, L_u) = \frac{M(V, L_u)}{L_u} = \frac{V}{(V^\theta + L_u^\theta)^{1/\theta}}. \]

Each worker decides whether to participate in the industrial labor market at the beginning of each period. Those who are already employed in the industrial sector can continue with their current job unless their employer lays them off or shuts down entirely. (They can also quit in order to move to the service sector or to search for other industrial sector jobs, although in equilibrium none find it optimal to do so.) Those not currently employed in the industrial sector - including those who just lost their jobs at contracting or exiting firms - can forego certain employment with a service sector firm in order to search for a higher-wage industrial sector job, but they risk remaining unemployed if they fail to match with an industrial sector producer.\(^7\) Those who end up unemployed subsist during the current period by home-producing services.

Each period, industrial sector firms decide whether to exit, fire some workers, maintain their existing work force, or hire workers. Firms that shed labor pay a firing cost \(c_f\) per worker dismissed, and they pay the workers they retain wages that are determined by Nash bargaining. Firms that post vacancies fill them at the rate \(\phi\), then they too bargain with their employees to determine wages. Finally, given the service sector technology (5), workers who opted for employment in the service sector are employed with certainty at the wage \(w_s = 1\), and workers who sought industrial sector jobs but failed to find them home-produce services at a wage of \(b\).

### 2.4 Timing of Events for Firms

Figure 1 describes timing of events for firms. A period has three stages. At the beginning, incumbents decide to exit or stay in the market. Continuing firms are hit by a productivity shock at the interim stage and thereafter make employment decisions. If a firm contracts, it commits to a wage schedule which leaves its workers indifferent between staying or leaving. If it expands, vacancies are posted and matching takes place. Thereafter the labor market closes and expanding firms bargain with all their workers, including new hires. At the end of the period production and consumption take place.

Entrants pay a sunk entry cost of \(c_e\) at the beginning of the period and draw their

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\(^7\)The notion that workers trade job security in a low wage sector for the opportunity to search in a higher wage sector traces back at least to the Harris and Todaro (1970) model.
initial productivity. Endowed with an initial workforce of $l_e > 0$ workers, they behave exactly like incumbent firms thereafter, with their interim state given by $(z, l_e)$. Next, we describe the firm’s problem in more detail.

### 2.5 Incumbent Firm’s Problem

Given the demand function (3) and the production function (6), any firm in state $(z, l)$ that sells some fraction $1 - \eta$ of its output domestically will generate home sales amounting to

$$R_H(z, l, \eta) = D^\frac{1}{\alpha} [(1 - \eta)zl^{\alpha}]^{(\alpha - 1)}. \quad (7)$$

Similarly, by exporting the remaining fraction $\eta$ such a firm will generate foreign sales of

$$R_F(z, l, \eta) = kD^\frac{1}{\alpha} \left[ \frac{\eta}{\tau_c} zl^{\alpha} \right]^{\alpha - 1},$$

where $D_F$ is the foreign demand parameter. We assume the home country is too small to influence foreign market aggregates, so $D_F$ exogenous to the model.

There are no start-up costs or adjustment costs associated with exporting, so firms choose $\eta$ each period to maximize their total current sales revenues, net of fixed exporting costs, $c_x$. The associated revenue function is

$$R(z, l) = \max_{\eta \in [0,1]} \{R_H(z, l, \eta) + R_F(z, l, \eta) - c_x I^x(z, l)\}, \quad (8)$$

where $I^x(z, l)$ is an indicator function that takes a value of unity when $\eta > 0$. Whether this occurs simply depends upon $zl^\alpha$, since the gains from foreign market participation...
increase monotonically with production. Given foreign market participation, the optimal fraction of output to export is given by

\[ \eta^o = \left( 1 + \frac{\tau_c \sigma^{-1} D_H}{k \sigma D_F} \right)^{-1}. \]  

Embedded in our general equilibrium model, this revenue function delivers a number of desirable features. First, it implies that for any given \((z, l)\), the marginal revenue product of labor is larger if the firm is an exporter. Thus productivity shocks induce larger adjustments in vacancy postings or firings when foreign markets are accessible. Second, this feature of the revenue function implies that revenue per unit input bundle is higher among exporters. Hence, our model explains the well-known association between revenue-based productivity measures and exporting, but it does so in a new way: factor market frictions cause exporters to have higher mark-ups.\(^8\) Third, since larger revenues mean more surplus to bargain over, exporters at a given \((z, l)\) pay higher relative wages than they would in a closed economy equilibrium. This result is consistent with the empirical finding that, controlling for employment, exporters pay their workers more (Bernard and Jensen (1999)). Fourth, since search frictions make marginal costs vary across firms with identical \(z\) values, our model explains why productive efficiency is a noisy predictor for exporting status.\(^9\) Finally, re-interpreting \(z\) shocks to be product appeal indices rather than efficiency indices, it explains why exporters manage to be larger than non-exporters, even though they charge higher prices and pay higher wages.\(^{10}\)

When choosing employment levels, firms weigh the associated revenue stream against wage costs, the effects of changes in \(l\) on the their continuation value, and current firing or hiring costs. To characterize the latter, let the cost of posting \(v\) vacancies for a firm of size \(l\) be

\[ C_h(l, v) = \left( \frac{c_h}{\lambda_1} \right) \left( \frac{v}{l \lambda_2} \right)^{\lambda_1}, \]

where \(c_h\) and \(\lambda_1 > 1\) are positive parameters.\(^{11}\) The parameter \(\lambda_2 \in [0, 1]\) determines

\(^8\)In support of this interpretation, De Loecker and Warzynski (2009) report evidence that mark-ups are higher among exporting firms. (They do not model pricing behavior.)

\(^9\)This fact has attracted some attention recently. Hallak and Sivadasan (2009) explain it by assuming that (1) firms differ in terms of both their quality and their productivity efficiency, and (2) exporting requires that firms meet a minimum quality standard.

\(^{10}\)Kugler and Verhoogen (2010) note that this pattern could alternatively be due to complementarities in production between worker ability and product quality.

\(^{11}\)This specification generalizes Nilsen et al. (2007), who set \(\lambda_2 = 1 - 1/\lambda_1\). See also Monika
the strength of scale economies in hiring. If $\lambda_2 = 0$, there are no economies of scale and the cost of posting $v$ vacancies is the same for all firms. On the other hand if $\lambda_2 = 1$, the cost of a given employment growth rate is the same for all firms. For any any $\lambda_2 > 0$, a given number of vacancies cost less for larger firms.

Firms in our model are large in the sense that cross-firm variation in realized arrival rates is ignorable. That is, all firms fill the same fraction $\phi$ of their posted vacancies. It follows that expansion from $l$ to $l'$ simply requires the posting of $v = \frac{l' - l}{\phi}$ vacancies, and we can write the cost of expanding from $l$ to $l'$ workers as

$$C_h(l, l') = \left( \frac{c_h}{\lambda_1} \right) \phi^{-\lambda_1} \left( \frac{l' - l}{l \lambda_2} \right)^{\lambda_1}.$$  

Clearly, when labor markets are slack, hiring is less costly because each vacancy is relatively likely to be filled.

When a firm reduces its workforce from to $l' < l$, it incurs firing costs equal to

$$C_f(l, l') = c_f(l - l').$$

All labor adjustment costs are in terms of the service good.\footnote{As is standard in the literature (see Ljungqvist (2002) for a review), we assume that firing costs take the form of a resource cost and are not pure transfers from firms to workers.} Note, however, that firing costs are proportional to the number of workers fired, so firms have no incentive to downsize gradually. When the firm exits, it is not liable for $c_f$. Also, as will be discussed below, it is possible that a firm will find itself in a position where the marginal worker reduces operating profits, but it is more costly to fire her than retain her.

Regardless of whether a firm is expanding, contracting, or remaining at the same employment level, we assume that it bargains with each of its workers individually and continuously. This ensures, as in Stole and Zwiebel (1996), Cahuc and Wasner (2000), and Cahuc, Marque, and Wasmer (2008) that all workers at a given firm are paid the same wage at a given point in time. Details of the resulting wage schedules are deferred to Section 2.7 below.

We now elaborate firms’ optimal employment policies within a period (see Figure 1). An incumbent firm enters the current period with the productivity $z$ and work...
force $l$ levels that were determined in the previous period. Immediately the firm decides whether to stay in business or to exit. If it stays, it proceeds to an interim stage in which it observes its current-period productivity realization $z'$. Then, taking stock of its updated state, $(z', l)$, the relevant wage schedules, and the sector-wide worker arrival rate, $\phi$, it chooses its current period work force, $l'$. If the firm decides to hire workers ($l' > l$), they are immediately available to produce output in the current period. If it fires workers ($l' \leq l$) it clears them from the payroll prior to production, although it incurs firing costs $C_f(l, l')$. Finally, revenues accrue and wages and other costs are paid at the end of the period.

Given the presence of search frictions, workers at hiring firms generate rents, and these are bargained over to determine wages. However, the marginal worker at a firing firm creates no rents and has no bargaining power. Hence expanding firms face different wage schedules from contracting or constant-employment firms, and current operating profits depend upon both $l$ and $l'$. More precisely, defining $w_h(z', l')$ to be the wage function faced by a hiring firm and $w_f(z', l')$ to be the wage function faced by a non-hiring firm, profits before labor adjustment costs are

$$
\pi(z', l, l') = \begin{cases} 
R(z', l') - w_h(z', l')l' - c_p & \text{if } l' > l, \\
R(z', l') - w_f(z', l')l' - c_p & \text{otherwise},
\end{cases}
$$

where $c_p$, the per-period fixed cost of operation, is common to all firms. Using (10), the beginning-of-period value of a firm in state $(z, l)$ is

$$
V(z, l) = \max \left\{ 0, \frac{1}{1 + r} E_{z'|z} \max_{l'} [\pi(z', l, l') - C(l, l') + V(z', l')] \right\},
$$

where the maximum of the term in square brackets is the value of the firm in the interim state (after it has realized its productivity shock), and

$$
C(l, l') = \begin{cases} 
C_h(l, l') & \text{if } l' > l, \\
C_f(l, l') & \text{otherwise}.
\end{cases}
$$

The solution to (11) implies an employment policy function,

$$
l' = L(z', l),
$$
an indicator function that distinguishes hiring and firing firms,

\[ I^h(z',l) = \begin{cases} 1, & \text{if } L(z',l) > l, \\ 0, & \text{otherwise.} \end{cases} \] (13)

\[ I^f(z',l) = \begin{cases} 1, & \text{if } L(z',l) < l, \\ 0, & \text{otherwise.} \end{cases} \] (14)

and an indicator function that characterizes firm’s continuation/exit policy,

\[ I^c(z,l) = \begin{cases} 1, & \text{if } V(z,l) > 0, \\ 0, & \text{otherwise.} \end{cases} \] (15)

### 2.6 Entry

In the steady state, a constant (endogenous) fraction \( \mu_{\text{exit}} \) of firms exits the industry. These firms are replaced by an equal number of entrants, who find it optimal to pay a sunk entry cost of \( c_e \) and create new firms. Upon creating their firms, these entrants acquire \( l_e > 0 \) workers and draw their initial productivity level from the ergodic distribution implied by \( h(z'|z) \), hereafter denoted \( f_e(z) \). The search costs for the initial \( l_e \) workers are included in \( c_e \). Thereafter entrants behave exactly like incumbent firms, with their interim state given by \( (z,l_e) \) - see Figure 1. So by the time they begin producing, most new entrants have adjusted their workforce (subject to search costs) in accordance with their initial productivity. Free entry implies that

\[ V_e = \int_z V(z,l_e)f_e(z)dz \leq c_e, \] (16)

which holds with equality if there is a positive mass of entrants, \( M \).

### 2.7 Worker’s Problem

Figure 2 presents the intra-period timing of events for workers. Consider first a worker who is employed by an industrial firm in state \( (z,l) \) at the beginning of the current period. This worker learns immediately whether her firm will continue operating. If it shuts down, she joins the pool of industrial job seekers (enters state \( u \)) in the interim stage. Otherwise, she enters the interim stage as an employee of the same firm that she worked for in the previous period. (No one voluntarily quits because, in equilibrium, firms always pay their workers at least their reservation wage.) Her firm then realizes its new productivity level \( z' \) and enters the interim
state \((z', l)\). At this point her firm decides whether to hire workers. If it expands its workforce to \(l' > l\), she earns \(w_h(z', l')\), and she is positioned to start the next period in state \((z', l')\). If the firm contracts or remains at the same employment level, she either loses her job and reverts to state \(u\) or she retains her job, earns \(w_f(z', l')\), and starts next period in state \((z', l')\). All workers at contracting firms are equally likely to be laid off, so each loses her job with probability \(p_f = (l - l')/l\).

Workers in state \(u\) are searching for industrial jobs. They are hired by entering and expanding firms that post vacancies. If they are matched with a firm, they receive the same wage as those who were already employed by the firm. If they are not matched, they support themselves by joining the informal sector and home-producing \(b \in [0, 1]\) units of the service good. At the start of the next period, they can choose to work in the service sector (enter state \(s\)) or look for a job in the industrial sector (remain in state \(u\)). Likewise, workers who start the current period in the service sector choose between continuing to work at the service wage \(w_s = 1\) and entering the pool of industrial job-seekers. As these workers have the option to choose either labor market, they are said to be in state \(o\).

![Figure 2 – Within-period Sequencing of Events for Workers](image-url)
We now specify the value functions for the workers in the interim stage. Going to the service sector generates an end-of-period income of 1 and returns a worker to the o state at the beginning of next period. Accordingly, the interim value of this choice is

\[ J^s = \frac{1}{1 + r}(1 + J^o), \]  

(17)

Searching in the industrial sector exposes workers to the risk of spending the period unemployed, supporting themselves by home-producing \( b \) units of the service good. But it also opens the possibility of landing in a high-value job. Since the probability of finding a match is \( \tilde{\phi} \), the interim value of searching for an industrial job is

\[ J^u = \left[ \tilde{\phi}EJ^e_h + \frac{(1 - \tilde{\phi})}{1 + r}(b + J^o) \right], \]

(18)

where \( EJ^e_h \) is the expected value of matching with a hiring firm to be defined below.

The value of the sectorial choice is \( J^o = \max\{J^s, J^u\} \) and, ruling out equilibria without service sector firms, workers must be equally attracted to both types of production:

\[ J^o = J^s = J^u. \]

(19)

Combined with (17), this condition implies that \( J^o, J^s, \) and \( J^u \) are all equal to \( 1/r \).

The expected value of matching with an industrial job, \( EJ^e_h \), depends on the distribution of hiring firms and the value of the jobs they offer. For workers who match with a hiring firm in the interim state \( (z', l) \), the interim period value is given by

\[ J^e_h(z', l) = \frac{1}{1 + r} [w_h(z', l') + J^e(z', l')], \]

(20)

where \( l' = L(z', l) \) and \( J^e(z', l') \) is the value of being employed at an industrial firm in state \( (z', l') \) at the start of the next period. Accordingly, the expected value of a match for a worker as perceived at the interim stage is

\[ EJ^e_h = \int_{z'} \int_l J^e_h(z', l)g(z', l)dldz', \]

(21)

where \( g(z', l) \) is the density of vacancies across hiring firms

\[ g(z', l) = \frac{v(z', l)\bar{f}(z', l)}{\int_{z'} \int_l v(z', l)\bar{f}(z', l)dldz'}. \]

(22)

Here \( v(z', l) = \mathcal{I}^h(z', l) [L(z', l) - l] / \phi \) gives the number of vacancies posted by a firm
in interim state \((z', l)\), and \(\tilde{f}(z', l)\) is the interim stage unconditional density of firms over \((z', l)\). (The latter density is generally distinct from the end-of-period stationary distribution of firms, \(\psi(z, l)\).)

It remains to specify the value of starting the period matched with an industrial firm, \(J^e(z, l)\), which appears in (20) above. The value of being at a firm that exits immediately is simply the value of being unemployed, \(J^u\). This is also the value of being at a non-hiring firm, since workers at these firms are indifferent between being fired and retained. Hence \(J^e(z, l)\) can be written as

\[
J^e(z, l) = I_c(z, l) E_{z'|z} \left\{ I_h(z', l) J^e_h(z', l) + [1 - I_h(z', l)] J^u \right\} + [1 - I_c(z, l)] J^u. \tag{23}
\]

### 2.8 Wage Schedules

We now characterize the wage schedules. Consider first a hiring firm. After hiring firms have posted their vacancies and matching has taken place, the labor market closes. Firms then bargain with their workers simultaneously and on a one-to-one basis, treating each worker as the marginal one. At this point vacancy posting costs are already sunk and workers who walk away from the bargaining table cannot be replaced in the current period. Similarly, if an agreement between firm and the worker is not reached, the worker remains unemployed in the current period. These timing assumptions create rents to be split between the firm and the worker.

As detailed in Appendix 1, it follows that the wage schedule for hiring firms with an end-of-period state \((z', l')\) is given by\(^{13}\)

\[
w_h(z', l') = (1 - \beta) b + \Gamma \cdot \Delta(z', l')(z')^{\frac{\sigma - 1}{\sigma}} (l')^{\frac{\sigma - 1}{\sigma}} [\theta^{(1-\alpha)}] - \beta P_f(z', l') c_f, \tag{24}
\]

where

\[
\Delta(z', l') = D_{lh}^\frac{1}{\sigma} [1 - \eta^\sigma I^x(z', l')]^{\frac{\sigma - 1}{\sigma}} + k D_{lc}^\frac{1}{\sigma} \tau_c^{\frac{\sigma - 1}{\sigma}} [\eta^\sigma I^x(z', l')]^{\frac{\sigma - 1}{\sigma}},
\]

and

\[
\Gamma = \frac{\alpha \beta (\sigma - 1)}{\sigma (1 - \beta) + \alpha \beta (\sigma - 1)}.
\]

In (24), \(\beta \in [0, 1]\) measures the bargaining power of the firm and \(P_f(z', l')\) is the probability of being fired next period. Worker in hiring firms get the marginal product of labor plus \((1 - \beta)\) share of their outside option, while part of the firing cost is passed to them as lower wages.\(^{14}\)

---

13This expression is analogous to equation (9) in Koeniger and Prat (2007).
14As in Bartelo and Caballero (1994), wages decline in firms’ employment \(l’\), holding productivity...
The marginal worker at a non-hiring firm generates no rents, so the firing wage just matches her reservation value (see Appendix 1)

\[ w_f(z', l') = rJ^u - [J^e(z', l') - J^u]. \] (25)

Three assumptions lie behind this formulation. First, workers who quit do not trigger firing costs for their employers. They thus enjoy no bargaining power when, at their reservation wage, they contribute nothing to their employer’s expected profit stream. Second, firms cannot use mixed strategies when bargaining with workers. Finally, workers who are fired are randomly chosen. The first assumption ensures that workers at contracting firms are paid no more than the reservation wage, and the remaining assumptions prevent firms from avoiding firing costs by paying less than reservation wages to those workers they wish to shed.

Importantly, \( w_f(z', l') \) does vary across firms, since those workers who continue with a firing firm may enjoy higher wages next period. This option to continue has positive value (captured by the bracketed term in (25)), so firing firms may pay their workers less than the flow value of being unemployed.

### 2.9 Equilibrium

Six basic conditions characterize our equilibrium. First, the distribution of firms over \((z, l)\) states reproduces itself each period through the Markov processes on \(z\), the policy functions (including hiring, firing, entry and exit), and the productivity draws that firms receive upon entry. Second, all markets clear: supply matches demand for services and for each differentiated good, where supplies are determined by employment and productivity levels in each firm. Third, the flow of workers into unemployment matches the flow of workers out of unemployment—that is, the Beveridge condition holds. Fourth, a positive mass of entrants replaces exiting firms every period so that free entry condition (16) holds with equality. Fifth, aggregate income matches aggregate expenditure, so trade is balanced. Finally, workers optimally choose the sector in which they are working or seeking work. Appendix 2 provides further details.
3 Quantitative Analysis

3.1 Pre- and Post-reform conditions in Colombia

To explore the implications of our model, we fit it to Colombian data. This country suits our purposes for several reasons. First, Colombia underwent significant trade liberalization during the late 1980s and early 1990s, reducing its average nominal tariff rate from 21.5% to 11.3% (Goldberg and Pavcnik, 2004). Second, Colombia also implemented labor market reforms in 1990 that substantially reduced firing costs.\textsuperscript{15} Finally, major changes in Colombian trading patterns and labor markets followed these reforms, suggesting that they may well have been important.

Key features of the Colombian economy during the pre- and post-reform period are summarized in Figure 3. (Due to data availability constraints, some series have gaps, and not all series cover the entire time period of interest.) The first panel shows the fraction of manufacturing establishments that were exporters (upper line), as well as the average share of output these exporters shipped abroad (lower line). Both ratios increased by about 250 percent from the 1980s to the 2000s. The second panel show job turnover rates among manufacturing plants, due both to expansion/contraction and entry/exit. This series went from an average of 18.5% during the pre-reform period 1978-1991 to 22% during the post-reform period 1993-1998. The third panel of Figure 3 shows the evolution of economy-wide unemployment rates. During the post-reform years 1991-1998, this series hovered around its 1980-90 average of 11%, but a financial crisis at the end of the decade pushed it sharply upward. The fourth panel shows the ratio of salaried workers to informal self-employed workers. Immediately after the reforms were implemented, this ratio began a sustained fall. Finally, the fifth panel shows that over the same time period, the Gini coefficient for Colombia rose from roughly 53% to roughly 58%. Though we have not graphed them, we note that similar patterns emerge from data on the share of income going to the top decile of the income distribution.

In sum, during the decade following reforms, Colombia registered marked increases in manufacturing trade, income inequality, and informality. It also showed a moderate increase in job turnover. We now investigate whether, in the context of our model, these changes can be attributed to the reductions in tariffs and firing costs that the

\textsuperscript{15}See Kugler (1999) for a summary of the Colombian Labor Market Reform of 1990. There were two components to the reform. First, as documented by Heckman and Pages (2004), severance payments were reduced. Second, beyond severance payments, these reforms also lowered the dead-weight cost associated with dismissals.
country implemented during the late 1980s and early 1990s.

**Figure 3** – Colombian Aggregates

3.2 Fitting the Model to Data

In fitting our model to Colombia, we use data from 1978-91 to approximate the pre-reform steady state. First, we estimate the parameters of the production function and the shock process using the panel of establishments covered by Colombia’s Annual Survey of Manufacturers. We then fix some standard parameters at values reported by previous studies. Finally, we calibrate the remaining parameters to match a wide set of moments related to the aggregate economy and firm-level behavior.
3.2.1 The Revenue Function and Productivity Process

To estimate the revenue function and the productivity process, we first use (3), (4) and (8) to write log revenues gross of fixed exporting costs as

\[
\ln R_{it} = d_H + I^x_{it} d_F + \frac{\sigma - 1}{\sigma} \ln z_{it} + \alpha \frac{\sigma - 1}{\sigma} \ln l_{it},
\]

where

\[
d_H = \ln \left[ D_H^\frac{1}{\sigma} (1 - \eta^o) \frac{\sigma - 1}{\sigma} \right],
\]

\[
d_F = \ln \left[ (k^\sigma D_F)^\frac{1}{\sigma} (\eta^o / \tau_c) ^{\frac{\sigma - 1}{\sigma}} e^{-d_H} + 1 \right],
\]

and \( I^x_{it} \) is an indicator for whether firm \( i \) is an exporter. Then assuming that \( \ln(z) \) follows an exogenous AR(1) process,

\[
\ln z_{it} = \rho \ln z_{i,t-1} + \epsilon_{it},
\]

we eliminate unobserved productivity shocks from (26) by quasi-differencing:

\[
\ln R_{it} = (d_H + I^x_{it} d_F) - \rho (d_H + I^x_{it-1} d_F) + \rho \ln R_{i,t-1} \]
\[
- \alpha \rho \left( \frac{\sigma - 1}{\sigma} \right) \ln l_{i,t-1} + \alpha \left( \frac{\sigma - 1}{\sigma} \right) \ln l_t + \frac{\sigma - 1}{\sigma} \epsilon_{it}.
\]

If we could obtain consistent estimates of the coefficients that appear on the right-hand-side observable variables, these would allow us to infer consistent estimates of \( d_H, d_F, \rho, \alpha \left( \frac{\sigma - 1}{\sigma} \right), \) and \( \sigma^2_{\epsilon} \). However, selection bias and simultaneity bias prevent us from consistently estimating (30) with ordinary least squares. The former problem occurs because by (15), firms choose whether to exit the market partly on the basis of their current productivity levels, so the \( \epsilon_{it} \) realizations observed for active producers are not random draws from the unconditional distribution of \( \epsilon \)'s. The latter problem occurs because firms’ current exporting decisions and employment levels are chosen after the current realization on \( \epsilon \) is observed, so \( \epsilon_{it} \) is correlated with both \( I^x_{it} \) and \( \ln l_{it} \). Appendix 3 develops a GMM estimator related to Olley and Pakes’ (1996) that deals with both problems.

Applying this estimator to the set of all Colombian manufacturing plants observed for at least three years during the pre-liberalization period 1982 and 1991, we obtain the results summarized in Table 3.\(^\text{16}\) Since \( \sigma \) is not identified separately from \( \alpha \),

\(^{16}\text{The data are annual observations on all manufacturing firms with at least 10 workers. They} \)
Table 1 – Estimates for the Revenue Function and Productivity Process

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.592</td>
<td>0.057</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.848</td>
<td>0.007</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>1.668</td>
<td>0.042</td>
</tr>
<tr>
<td>$d_H$</td>
<td>1.682</td>
<td>0.047</td>
</tr>
<tr>
<td>$d_F$</td>
<td>0.213</td>
<td>0.004</td>
</tr>
</tbody>
</table>

we fixed this parameter at a value typical of the literature: $\sigma = 5$. All remaining parameters are estimated with considerable precision. It should be noted, however, that the estimates are sensitive to choice of the instrument set, and to the weights we used on different types of workers-managers, technicians, skilled laborers, unskilled workers, and apprentices-when constructing the number of “effective” workers. We have chosen instruments and weights that yield $\alpha$ and $\rho$ values typical of the literature so, while the standard errors give a sense for fit, they should not be used for statistical inference.

3.2.2 Remaining Parameters

We next fix several parameters using external sources. First, the real borrowing rate in Colombia fluctuated around 15 percent between late 1980s and early 2000s, so we set $r$ to be 0.15 (Bond et al, 2008). Second, as is common in the labor literature, we give equal bargaining power to firms and workers, setting $\beta = 0.5$. Finally, we set iceberg trade costs at $\tau_c - 1 = 1.50$ following Eaton and Kortum (2002) who find that the tariff equivalent of iceberg costs falls between 123 percent and 174 percent. This $\tau_c$ value, along with our estimates for $d_F$ and $d_H$ in Table 3, implies $D_H$ and $k^\sigma D_F$. Equations (27), (28), and (9) imply $\exp(d_F) = (1 - \eta_0)^{-1}$, so we can impute $\eta_0$ from the estimated value of $d_F$. Substituting this value into $\exp(d_H) = D_H^{1/\sigma}(1 - \eta_0)^{(\sigma-1)/\sigma}$ yields $D_H$, given $\sigma$ and the estimated value of $d_H$. Finally, given a value for $\tau_c$, $k^\sigma D_F$ follows from (9).

were collected by Colombia’s National Statistics Department (DANE) and cleaned as described in Roberts (1996). Given that fixed capital and intermediate inputs do not appear in our model, we define revenue to be the value of output net of intermediate input and capital costs. Annual capital costs are 10 percent of the book value of firms’ capital stocks.

17The weights used for reported estimates are based on cross-plant mean wage premiums for each type of employee, relative to unskilled workers. Weighting means (using plant size as weights) yields a larger $\alpha$ value, although it has little effect on $\rho$.

18Equations (27), (28), and (9) imply $\exp(d_F) = (1 - \eta_0)^{-1}$, so we can impute $\eta_0$ from the estimated value of $d_F$. Substituting this value into $\exp(d_H) = D_H^{1/\sigma}(1 - \eta_0)^{(\sigma-1)/\sigma}$ yields $D_H$, given $\sigma$ and the estimated value of $d_H$. Finally, given a value for $\tau_c$, $k^\sigma D_F$ follows from (9).
Table 2 – Parameters Fixed Before Simulating Moments

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.592</td>
<td>Production function</td>
<td>GMM estimate (Table 1)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.848</td>
<td>Persistence of ( z ) process</td>
<td>GMM estimate (Table 1)</td>
</tr>
<tr>
<td>( \sigma_z )</td>
<td>1.291</td>
<td>Std. dev. of shocks to ( z )</td>
<td>GMM estimate (Table 1)</td>
</tr>
<tr>
<td>( k^aD_F )</td>
<td>635.6</td>
<td>Foreign demand level</td>
<td>GMM estimates (Table 1)</td>
</tr>
<tr>
<td>( \tau_c )</td>
<td>2.5</td>
<td>Iceberg trade costs</td>
<td>Eaton and Kortum (2003)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.5</td>
<td>Bargaining power</td>
<td>assumed (literature)</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>5</td>
<td>Elasticity of substitution</td>
<td>assumed (literature)</td>
</tr>
<tr>
<td>( r )</td>
<td>0.15</td>
<td>Discount rate</td>
<td>Bond, et al (2008)</td>
</tr>
</tbody>
</table>

cost of operation, \( c_p \), the fixed cost of exporting, \( c_x \), the value of informal sector production, \( b \), the firing cost in terms of service sector goods, \( c_f \), the initial size of new firms, \( l_e \), the share of differentiated goods in total expenditures, \( \gamma \), the parameters of the vacancy cost function, \( (c_h, \lambda_1, \lambda_2) \), the elasticity of matching function \( \theta \), and the cost of creating a new firm, \( c_e \). Our final step in fitting the model is to calibrate these parameters using 17 targets: the firm exit rate, the job turnover rate, the fraction of firms that export, the unemployment rate plus the informality rate, firing cost (in terms of annual wages), the autocorrelation of firms’ employment levels, correlation between firms’ productivity and employment, the employment growth rates among expanding firms at the different quintiles of the size distribution, the share of workers in the service sector, and the size of distribution of firms.\(^\text{19}\) Our solution algorithm is summarized in Appendix 4.

While it is not possible to associate individual parameters with individual statistics, experiments do suggest that particular statistics play relatively key roles in identifying particular parameters. First, the fraction of firms that export is sensitive to fixed exporting costs, \( c_x \), and the rate of firm turnover is very responsive to the per-period fixed costs of operating a business, \( c_p \). Second, the quintile-specific job growth rates and the aggregate labor turnover rate are responsive to the parameters

\(^{19}\)We do not calibrate to measures of wage dispersion because it is not possible for us to completely control for differences in worker characteristics when constructing data-based measures of wage heterogeneity and arrive at a measure of residual wage inequality. Interestingly, however, as will become evident in our discussion of policy experiments, our model economy is able to generate a high level of wage inequality within a labor search framework. This is traceable to the low job finding rate in our benchmark economy (about 5% per year), since with a low job finding rate workers are willing to take low wages. As noted by Hornstein, Krusell and Violante (2010), the standard search models deliver low wage inequality (compared to data) when they are calibrated to the high job finding rates observed in the U.S. Note that unemployment in the model corresponds to unemployment and informality in the data which justifies the low level of the calibrated job finding rate.
Table 3 – Calibration: Data-based versus Simulated Statistics*

<table>
<thead>
<tr>
<th>Industry-wide Statistics</th>
<th>Data</th>
<th>Model</th>
<th>Employment Growth Rates, by Quintile</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm exit rate</td>
<td>0.091</td>
<td>0.083</td>
<td>&lt; 20th percentile</td>
<td>0.317</td>
<td>0.297</td>
</tr>
<tr>
<td>Job turnover</td>
<td>0.216</td>
<td>0.213</td>
<td>20th – 40th percentile</td>
<td>0.217</td>
<td>0.201</td>
</tr>
<tr>
<td>Share of firms exporting</td>
<td>0.117</td>
<td>0.115</td>
<td>40th – 60th percentile</td>
<td>0.191</td>
<td>0.163</td>
</tr>
<tr>
<td>Unemployment+informality rate</td>
<td>0.278</td>
<td>0.297</td>
<td>60th – 80th percentile</td>
<td>0.163</td>
<td>0.156</td>
</tr>
<tr>
<td>Share of workers in S sector</td>
<td>0.550</td>
<td>0.581</td>
<td>&gt; 80th percentile</td>
<td>0.123</td>
<td>0.115</td>
</tr>
<tr>
<td>$corr(\ln(l), \ln(l'))$</td>
<td>0.95</td>
<td>0.96</td>
<td>20th percentile cutoff</td>
<td>13</td>
<td>10.63</td>
</tr>
<tr>
<td>$corr(\ln(z), \ln(l'))$</td>
<td>0.59</td>
<td>0.59</td>
<td>40th percentile cutoff</td>
<td>20</td>
<td>19.32</td>
</tr>
<tr>
<td>$corr(\ln(z), \ln(l))$</td>
<td>0.57</td>
<td>0.60</td>
<td>60th percentile cutoff</td>
<td>34</td>
<td>32.79</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>80th percentile cutoff</td>
<td>75</td>
<td>78.53</td>
</tr>
</tbody>
</table>

*The firm exit rate, quintile specific job turnover rates, and the fraction of firms that export are calculated from Colombian plant level data for the pre-liberalization period, 1978-91. These data were collected by the Colombian National Administrative Department of Statistics (DANE) in its Annual Manufacturer Survey (EAM), which covers all establishments with at least 10 workers. The statistics $corr(\ln(l), \ln(l'))$, $corr(\ln(z), \ln(l'))$ and $corr(\ln(z), \ln(l))$ are based on the same data base and time period, using the technology estimates in Table 3 to calculate $z$. The unemployment rate is taken from Inter-American Development Bank (2004), and is based on DANE’s biennial National Household Survey (ENH). The share of workers in the service sector and the informality rate are also calculated from the ENH, defining an informal sector worker to be someone who does not pay social security, is self-employed, has no employees, and is doing neither professional/technical nor managerial work.

of the vacancy cost function $(c_h, \lambda_1, \lambda_2)$, with cross-quintile differences governed by the scale economies parameter, $\lambda_2$ and (for the smallest quintile) the initial size of new firms, $l_e$. Third, the share of workers in the service sector responds to the share of service goods in total expenditures, $\gamma$. Fourth, the unemployment/informality rate is very responsive to the productivity of informal sector workers, $b$. Finally, the firing cost parameter $c_f$ and the elasticity of matching function $\theta$ play key roles in shaping the size distribution.

Table 3 reports the data-based statistics we use for calibration and their model-based simulated counterparts. Although we are using 11 parameters to try to match 17 statistics, the model does a nice job of fitting the data overall. In particular, it captures the size distribution of firms, the contributions of firm entry/exit and intra-firm size adjustments to overall job turnover, the persistence in employment levels, the overall unemployment rate, and higher job turnover rate among small firms.

Note that we use the sum of unemployment and informal self-employment as our data target. Since Colombia does not have an unemployment insurance system, it is common for unemployed workers to be self-employed at jobs with low entry costs (such as street vending) while searching for a salaried job. As a result, flows from informal self-employment into formal employment are substantial compared to flows

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$\cdot$The metric of fit we used was the average $|1 - Y_i/X_i|$ where $X_i$ is the $i^{th}$ data-based statistics and $Y_i$ is the corresponding model-based statistic. At its minimized value, this statistic was 0.061.
Table 4 – Calibrated Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_p$</td>
<td>31.41</td>
<td>Fixed cost of operation</td>
</tr>
<tr>
<td>$c_h$</td>
<td>7.82</td>
<td>Scalar, vacancy cost function</td>
</tr>
<tr>
<td>$c_x$</td>
<td>19.92</td>
<td>Fixed exporting cost</td>
</tr>
<tr>
<td>$b$</td>
<td>0.73</td>
<td>Value of home production</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>2.20</td>
<td>Convexity, vacancy cost function</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.35</td>
<td>Scale effect, vacancy cost function</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.49</td>
<td>Share of $Q$ goods in total spending</td>
</tr>
<tr>
<td>$l_e$</td>
<td>3.10</td>
<td>Initial size of entering firms</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1.41</td>
<td>Elasticity of matching function</td>
</tr>
<tr>
<td>$c_e$</td>
<td>210.9</td>
<td>Entry cost for new firms</td>
</tr>
<tr>
<td>$c_f$</td>
<td>1.21</td>
<td>Firing cost</td>
</tr>
</tbody>
</table>

from unemployment.\textsuperscript{21}

Table 4 reports the parameter values associated with the calibration. Those expressed in monetary units are measured in terms of the 1990 average annual wage for a service sector worker, taken from the annual household survey. This figure amounted to roughly $4,500 current US dollars. Accordingly, our model implies that the costs of creating a new firm are about $948,420, the fixed costs of operating a business amount to about $141,345, and the fixed costs of exporting are about $85,654. Note also that those who end up working in the informal sector take about a 27 percent wage cut relative to what they could have earned if they had committed to working for a service sector firm. The parameters of the vacancy cost function imply both short-run convexities ($\lambda_1 = 2.20$) and modest scale economies ($\lambda_2 = 0.35$).\textsuperscript{22} The elasticity of matching function, $\theta = 1.41$, is not far from the value of 1.27 that den Haan et al (2000) obtain in calibrating their model to the U.S. economy. Finally, the firing cost is about 60% of the average yearly wage in manufacturing.

\textsuperscript{21}Bosch and Maloney (2007) document this type of gross worker flow in the presence of informal labor markets in the context of Mexico, a country with a similar institutional setup.

\textsuperscript{22}Our of $\lambda_1$ is consistent with the available evidence on hiring cost convexities (e.g. Merz and Yashiv (2007), and Yashiv (2006)). We also come close to satisfying the relationship $\lambda_2 = 1 - 1/\lambda_1$ implied by Nilsen et al.’s (2007) specification.
3.3 Simulated Effects of Openness and Firing Costs

We are now prepared to examine the effects of trade reforms in our calibrated model. To do so, we first simulate each policy reform in isolation, then we consider their combined effects. Since the tariff reductions that Colombia implemented are not sufficient to explain the observed increase in trade flows during the post-reform period, we also consider a reduction in iceberg trade costs that, combined with the other reforms, replicates this increase. This decrease in trade costs can be interpreted to approximate the effects of greater openness among Colombia’s trading partners and general reductions in the costs of international commerce. Table 5 and Figures 4-7 summarize these experiments; we now turn to their interpretation.

Table 5 – Effects of Trade Costs and Firing Costs

<table>
<thead>
<tr>
<th></th>
<th>Base Case</th>
<th>Reductions in</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Tariffs</td>
</tr>
<tr>
<td>Tariffs ($τ_m$)</td>
<td>1.21</td>
<td>1.11</td>
</tr>
<tr>
<td>Iceberg costs ($τ_c$)</td>
<td>2.50</td>
<td>2.50</td>
</tr>
<tr>
<td>Firing costs ($c_f$)</td>
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<td>1.21</td>
</tr>
<tr>
<td>Employment Growth Rate by Quintile</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&lt; 20th percentile</td>
<td>0.297</td>
<td>0.299</td>
</tr>
<tr>
<td>20th – 40th percentile</td>
<td>0.201</td>
<td>0.209</td>
</tr>
<tr>
<td>40th – 60th percentile</td>
<td>0.163</td>
<td>0.144</td>
</tr>
<tr>
<td>60th – 80th percentile</td>
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<tr>
<td>&gt; 80th percentile</td>
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<td>0.121</td>
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<td>11.07</td>
</tr>
<tr>
<td>40th percentile</td>
<td>19.32</td>
<td>21.15</td>
</tr>
<tr>
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</tr>
<tr>
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<td>Aggregates relative to base case</td>
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</tr>
<tr>
<td>Average firm size</td>
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</tr>
<tr>
<td>Share of firms exporting</td>
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<td>1.146</td>
</tr>
<tr>
<td>Exit rate</td>
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<tr>
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</tr>
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<td>Log 90-10 wage ratio</td>
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</tr>
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<td>Share labor in Q sector</td>
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</tr>
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<td>Real wage dispersion, Q sector</td>
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</tr>
<tr>
<td>Real average wage, Q sector</td>
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<td>1.006</td>
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<td>Real average wage, S sector</td>
<td>1.000</td>
<td>1.014</td>
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3.3.1 Tariff Reductions

Consider first Colombia’s reduction in import tariffs from 21 percent to 11 percent (Table 5, column 2, and first panel of Figures 4 and 5). In the context of our model, this reform puts downward pressure on the domestic sales of $Q$-sector firms. But it also induces a real currency devaluation through the balanced trade condition, thereby increasing the optimal export share $\eta$ and raising the fraction of firms that export by 15 percent. This export expansion is concentrated among moderately large firms, so the cumulative firm size distribution shifts rightward.

As these changes in product markets occur, a number of forces link tariff rates to labor market outcomes. First, the shift in the size distribution concentrates worker at employers that make relatively modest percentage-wise adjustments in their employment levels. Second, trade liberalization moves the threshold output level for exporting, and thus changes the number and type of firms that adjust their exporting status in response to productivity shocks. This matters because firms that cross the exporting threshold enjoy larger rents and make larger percentage-wise employment adjustments than they would have if exporting had never been an attractive option. Third, and finally, tariff reductions change the distribution of rents across firms in different states, inducing associated adjustments in the wage distribution through the wage bargaining game.

Our simulations indicate that the first effect on job security was strong enough to offset the second, and thus Colombia’s trade liberalization did not in itself contribute to the long-run increases in unemployment and informality that emerged during the post-reform period. Nonetheless, we find that distributional effects were also important: trade liberalization increased the fraction of jobs at exporting firms with large rents, driving up real wage dispersion by nearly 7 percent. Thus, our model provides a structural interpretation for the association between openness and inequality in Latin America documented by Goldberg and Pavcnik (2007).  

3.3.2 Firing Costs

We next investigate the effects of firing cost reductions on job turnover and wage inequality. Heckman and Pages (2004) calculate that severance payments in Colombia declined from 17 months of wages to 13 months of wages after the reforms. Assuming

\footnote{This association between openness and wage inequality appears in other recent trade models with heterogeneous firms and rent-sharing. Relevant references include Helpman and Itskhoki (2010), Helpman et al. (2010), Egger and Kreickemeier (2007), Amiti and Davis (2008), Davis and Harrigan (2007), Felbermayr et al. (2008).}
that firing costs followed a similar pattern, we consider a reduction of about 25 percent in firing cost (Table 5, column 3, and second panel of Figures 4 and 5).

As we have stressed, the concentration of employment among small firms increases job turnover, other things equal. Here this effect is compounded by the well-known direct impact of firing cost reductions on job security (e.g., Ljungqvist (2002) and Mortensen and Pissarides (1999)), leading to a total increase in the turnover rate of 5.5 percent.

Relative to the pre-reform equilibrium, we find that large, inefficient firms shed labor. This is shown in Figure 6, which depicts the absolute changes in employment

Figure 4 – Effect of Trade and Labor Market Reforms on Firm Size Distribution

Figure 5 – Effect of Trade and Labor Market Reforms on Job Turnover
levels, $L(z', l)$, associated with lower firing costs. This downsizing of poor performers reduces the average firm size declines by 8.3 percent.

Lower firing costs also lead to more rents among expanding firms, increasing the average wage by about 4%. This entices more workers to search for $Q$-sector jobs, as in Harris and Todaro (1970), and in combination with the higher rate of job turnover it drives up the rate of informality/unemployment by 2.2%. Therefore, taken by itself, Colombia’s labor market reform go part way toward explaining the rising unemployment and informality the country experienced during the post-reform years.

![Figure 6 – Change in Employment Policy](image)

### 3.3.3 Trade Costs

Table 1 reports that the fraction of Colombian firms that exported increased by 260% during the post-reform period, but Table 5 indicates that tariff reductions should only have increased this fraction 15%. To reconcile our simulations with Colombia’s post-reform experiences, we could increase the elasticity of demand, $\sigma$, reduce the fixed costs of exporting, $f$, or reduce iceberg transport costs, $\tau$. We focus our analysis on a decline in iceberg trade costs because it would require an implausibly high $\sigma$ value to induce a tripling of exporters, and because lowering $c_x$ sufficiently to induce a 260% expansion results in implausibly small export shipments per firm. In contrast, the required 30% decline in trade frictions over the course of a decade seems plausible, given the greater openness of Colombia’s trading partners, and the general reductions in shipping costs and improvements in global communications that took
place during this time period.

Not surprisingly, a 30% reduction in $\tau$ affects the size distribution more dramatically than cutting tariffs by 11 percentage points (Table 5, column 4, and third panel of Figure 4). As predicted by Melitz (2003), small firms become relatively scarce while the top end of the size distribution shifts rightward. Thus jobs become concentrated at relatively stable firms. This alone would put downward pressure on the job turnover rate, but the increase in openness also results in more firms crossing the exporting threshold in response to productivity shocks, and unlike in the tariff experiment, the associate increase in hiring and firing rates is enough to offset the size distribution effect.

Finally, as with the tariff reduction, falling trade costs increase profits at firms with relatively low marginal costs and create greater rents to be bargained over. Wage dispersion rises 13% relative to the base case in consequence. The average real wage in the differentiated product sector also rises 18%, primarily because lower trade costs reduces the price index for differentiated goods. (For this reason the real service sector wage rises too.) So greater integration with foreign markets goes some way toward explaining the rising inequality that Colombia experienced in the post 1995 period.

3.3.4 Post-Reform Economy

The last column of Table 5 reports the combined effects of greater openness and reduced firing costs, i.e. the reform package that Colombia implemented in the early 1990s and the change in trade costs that occurred for other reasons. Since all of these changes push wage inequality and real average wages upward, both consequences come through clearly. On the other hand, while reductions in firing costs increase job turnover, openness has little effect on this aggregate, and the net effect is modest. Similar comments apply concerning the steady state unemployment/informality rate, which also increases slightly. Overall, then, stripping away the effects of macro shocks and Colombia's financial crisis, our simulations imply that the policy reforms implemented in the early 1990s improved wages on average at the cost of more inequality.

Clearly, our simulations do not completely predict post-reform labor market outcomes in Colombia. In particular, while experiments predict a slight increase in unemployment/informality, the actual change in informality was quite substantial. We infer that other forces were also in play, including possibly the interaction of comparative advantage effects and skill-biased technological change with job-specific human capital.
3.3.5 Welfare Effects of the Reforms

Overall, our simulations imply that the reforms increase average welfare by about 19%. All service sector workers share the same wage and future employment prospects, so the net effects on their welfare is summarized by the real wage effects reported in Table 5. However, some $Q$-sector workers were affected much more dramatically than others, and their fates depended upon their employer’s characteristics.

To summarize these $Q$-sector welfare effects, Figure 7 presents four graphs, each depicting the percentage change in $J^e(z,l)$ associated with a particular experiment. (Recall that $J^e(z,l)$ is the value of starting a period matched with a firm in state $(z,l)$.) The first panel depicts the effects of reducing tariffs from $\tau_m = 1.21$ to $\tau_m = 1.11$, the second panel depicts the effects of reducing firing costs from $c_f = 1.6$ to $c_f = 1.2$, the third panel depicts the effects of reducing iceberg costs from $\tau_c = 2.50$ to $\tau_c = 1.73$, and the last panel depicts the net effect of all these changes when they are simultaneously implemented.$^{24}$

Other things held fixed, the net gain to workers from reducing $\tau_m$ reflects both the effect on their wages and the change in the consumer price index, $P^\gamma$. Most workers come out ahead, but the workers at small, low-productivity firms do a bit worse. These firms face greater import competition after the tariff reduction, but they are too small to take advantage of the associated depreciation by exporting. Note

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$^{24}$The units in these figures are constructed to correspond to the quantiles reported in Table 5, so the grids for employment and productivity are aggregated to quantiles. Within each quantile, the height is a weighted average of the values at the different states, with weights given by the density of employment.
also that the wage effect is a bit stronger at large, low productivity firms because these firms are likely to draw the largest productivity shocks in the future and thus perceive relatively high future rents from the marginal worker. However, large firms are typically large because they have drawn a series of favorable productivity shocks, so this corner of state space is very sparsely populated.

Reductions in firing costs \( c_f \) also make workers more attractive to most employees, although they don’t do much for workers at small, low productivity firms. The reason is that firing costs are irrelevant for exiting firms, and these firms are relatively likely to exit.

The largest welfare effects come from our 30% reduction in trade costs, \( \tau_c \). This form of globalization strongly reduces the price of tradable goods, generating substantial welfare effects for all consumers. Nonetheless, the gains vary considerably across employers, with the biggest welfare increases coming at the firms that benefit most from exporting—that is, those with high productivity and many employees.

4 Conclusion

In Latin America, globalization and labor market reforms have been associated with less job security, more wage inequality, and more informality. We formulate a dynamic structural model that explains these patterns of association as a consequence of interactions between the policy reforms, idiosyncratic productivity shocks, exporting incentives, and scale economies in hiring workers. Simulations of our model imply that by themselves, tariff reductions are unlikely to have been the main reason that Colombia experienced deteriorating labor market conditions during the 1990s. However, the combined effects of reductions in firing costs and globalization go some way toward explaining observed increases in job turnover and wage dispersion.

In addition to providing a lens through which to interpret recently-observed changes in Latin American labor markets, our paper makes several methodological contributions. First, it generalizes the representation of labor markets with multi-worker firms developed by Bertola and Caballero (1994) to an open economy setting with fully articulated product markets, multiple sectors, and continuous Markov processes for productivity shocks. Second, it demonstrates how to quantify some welfare and distributional effects of openness and firing costs that have not previously been explored.
Appendix 1: The Wage Functions

**Hiring Wages** In order to characterize wages in hiring firms, we first determine the total surplus for a firm and a worker that are matched in the end-of-period state \((z', l')\). At the time of bargaining, the surplus that the marginal worker generates for a firm is given by

\[
\Pi_{\text{firm}}(z', l') = \frac{1}{1 + r} \left[ \frac{\partial \pi(z', l')}{\partial l'} + \frac{\partial V(z', l')}{\partial l'} \right].
\]

Note that at the time of bargaining, the vacancy posting and matching process are over and the costs of vacancy postings are sunk. As a result, if the bargaining fails, the firm is simply left with less workers. Thus we only use the relevant part of the profit function for hiring firms, i.e. when \(l' > l\) in (10), denoted by \(\pi(z', l')\). The surplus that a marginal worker generates consists of three parts: the current increase in the firms’ profits, i.e. marginal revenue product net of wages, and the increment to the value of being in state \((z', l')\) at the start of the next period. If the firm does not exit next period, i.e. if \(V(z', l') > 0\), the marginal worker will have a positive only if the firm expands. Otherwise, the firm will incur the dismissal cost \(c_f\). If the firm exits, its expected marginal value from the current marginal hire will be zero.

Similarly, the surplus for the marginal worker who is matched by a hiring firm in the end-of-period state \((z', l')\) is

\[
\Pi_{\text{work}}(z', l') = \frac{1}{1 + r} \left[ w_h(z', l') + J_e(z', l') - (b + J^o) \right],
\]

where the worker enjoys \(w_h(z', l')\) in the current period, and starts next period in a firm with the beginning-of-period state \((z', l')\). Since at the time of bargaining the vacancy posting and matching process are over, if the bargaining fails, the worker is unemployed this period and starts next period in state \(o\).

The worker and firm split the total surplus by Nash bargaining where the bargaining power of the firm is given by \(\beta\):

\[
\beta \Pi_{\text{firm}}(z', l') = (1 - \beta) \Pi_{\text{work}}(z', l').
\]

Wages are thus determined as a solution to the following equation

\[
\beta \left[ \frac{\partial \pi(z', l')}{\partial l'} + \frac{\partial V(z', l')}{\partial l'} \right] = (1 - \beta) \left[ w_h(z', l') + J_e(z', l') - (b + J^o) \right].
\]
Note that we cannot rule out the case in which a firm hires in the current period and exists at the beginning of next period. The bargaining outcome depends on the exit vs. continuation decision which is known by the time of bargaining. We analyze these two cases separately.

a) Exiting Firms: If the firm is going to exit next period, i.e. $I^c(z', l') = 0$, we have $\partial V(z', l')/\partial l' = 0$ and $J^c(z', l') = J^u$ from the definition of $J^e$. In this case, $\partial V(z', l')/\partial l'$ cancels with $J^c - J^o$ in (31) since $J^o = J^u$ in equilibrium. We are left with

$$\beta \frac{\partial \pi(z', l')}{\partial l'} = (1 - \beta)[w_h(z', l') - b].$$

Using the definition of $\pi(z', l')$ from (10), and rearranging terms, equation (32) becomes

$$\frac{\partial w_h(z', l')}{\partial l'} \beta l' + w_h(z', l') - \beta \frac{\partial R(z', l')}{\partial l'} - (1 - \beta)b = 0,$$

which is the same as equation (10) in Bertola and Garibaldi (2001). Substituting $\partial R(z', l')/\partial l'$ from (8), and solving the differential equation, the hiring wage schedule for next-period exiters is given by

$$w_h(z', l') = (1 - \beta)b + \Gamma \cdot \Delta(z', l')(z')^{\frac{\sigma-1}{\sigma}}(l')^{-[\phi^\tau + (1 - \phi^\tau)])},$$

where

$$\Delta(z', l') = D^\frac{1}{\sigma}k[1 - \eta^o I^x(z', l')]^{\frac{\sigma-1}{\sigma}} + D^\frac{1}{\sigma}k^\tau_c \eta^o I^x(z', l')]^{\frac{\sigma-1}{\sigma}};$$

and

$$\Gamma = \frac{\alpha \beta (\sigma - 1)}{\sigma (1 - \beta) + \alpha \beta (\sigma - 1)}.$$

b) Continuing Firms: In this case, we have $V(z', l') > 0$. There is an expected gain from keeping the marginal worker because of the possibility of further hiring next period. Expected gain of the worker in the beginning of next period (when she still has a chance to leave the firm and search) is $J^e(z', l') - J^u$. The pair shares the expected gains, i.e $J^e(z', l') - J^u$ cancels with the expected gain of the firm in (31). In event of a contraction, however, the firm cannot enforce contracts that stipulate laid-off workers to pay their share of firing costs. Thus, expected firing costs, $P_f(z', l')c_f$, is subtracted from firm surplus in the current
period:
\[ \beta \left[ \frac{\partial \pi(z', l')}{\partial l'} - P_f(z', l')c_f \right] = (1 - \beta)[w_h(z', l') - b]. \]

Conditional on firing taking place, the possibility of losing one’s job, \( p_f(z', l) \), is
\[ p_f(z', l) = \frac{l - L(z', l)}{l}. \]

The probability of being fired next period is then given by
\[ P_f(z', l') = \int I(z'', l')p_f(z'', l')h(z''|z')dz. \]

The wage schedule for expanding firms which will stay in the market next period is given by
\[ w_h(z', l') = (1 - \beta)b + \Gamma \cdot \Delta(z', l')(z')^{\frac{\sigma - 1}{\sigma}}(l')^{\frac{\sigma}{\sigma + 1}} - \beta P_f(z', l')c_f. \]

**Firing Wages** To derive the firing wage schedule, we begin by writing the value of employment at a firing firm in the interim stage as
\[ J^*_f(z', l) = \frac{1}{1 + r} \left[ p_f(z', l)((1 + r)J^u) + (1 - p_f(z', l)) (w_f(z', l') + J^e(z', l')) \right], \]
where \( l' = L(z', l) \). This expression reflects the fact that workers who are not fired are paid just enough to retain them. Next we note that, since workers are indifferent between staying and leaving
\[ w_f(z', l') + J^e(z', l') = (1 + r)J^u, \]
and the wage schedule faced by firing firms can be written as
\[ w_f(z', l') = rJ^u - [J^e(z', l') - J^u]. \]
Appendix 2: Steady State Equilibrium

A steady state equilibrium for a small open economy consists of a measure of domestic differentiated goods \( N_H \), an exact price index for composite good \( P \), an aggregate quantity index for composite good \( Q \), aggregate income \( I \), a measure of workforce in services \( L_S \), a measure of unemployed workers in differentiated goods sector \( L_u \), unemployment rate in differentiated goods sector \( \mu_u \), job finding rate \( \tilde{\phi} \), vacancy filling rate \( \phi \), the exit rate \( \mu_{exit} \), the measure of entrants \( M \), the value functions and associated policy functions \( \mathcal{V}(z, l) \), \( L(z, l) \), \( \mathcal{I}^h(z, l) \), \( \mathcal{I}^f(z, l) \), \( \mathcal{I}^e(z, l) \), \( J^o \), \( J^u \), \( J^e \), and \( J^c \); the wage schedules \( w_h(z, l) \) and \( w_f(z, l) \), exchange rate \( k \), and end-of-the period and interim distributions \( \psi(z, l) \) and \( \tilde{\psi}(z, l) \) such that

a) **Steady State Distributions:** In equilibrium, \( \psi(z, l) \) and \( \tilde{\psi}(z', l) \) reproduce themselves through the Markov processes on \( z \), the policy functions and the productivity draws upon entry. In order to define the interim distribution, \( \tilde{\psi}(z, l) \), let \( \tilde{\psi}(z', l) \) be the interim frequency measure of firms defined as

\[
\tilde{\psi}(z', l) = \begin{cases} 
\int_z h(z'|z)\psi(z, l)\mathcal{I}^c(z, l)dz & \text{if } l \neq l_e \\
 f_e(z') + \int_z h(z'|z)\psi(z, l)\mathcal{I}^c(z, l)dz & \text{if } l = l_e 
\end{cases}
\]

Then, \( \tilde{\psi}(z', l) \) is given by

\[
\tilde{\psi}(z', l) = \frac{\tilde{\psi}(z', l)}{\int_{z'} \int_l \tilde{\psi}(z', l)dz'dl},
\]

while the end-of-the period distribution is

\[
\psi(z', l') = \frac{\int_l \tilde{\psi}(z', l)\mathcal{I}_{(L(z', l), l')}dl}{\int_{z'} \int_l \tilde{\psi}(z', l)\mathcal{I}_{(L(z', l), l')}dz'dl},
\]

where \( \mathcal{I}_{(L(z', l), l')} \) is an indicator function with \( \mathcal{I}_{(L(z', l), l')} = 1 \) if \( L(z', l) = l' \).

b) **Market Clearance in Sector \( S \):** Demand for the \( S \)-sector goods comes from two sources: consumers spend a \( (1 - \gamma) \) fraction of aggregate income \( I \) on it, and firms demand it to pay their fixed operation costs, fixed exporting costs, labor adjustment and entry costs. The average labor adjustment cost is given by

\[
\tau = \int_z \int_l C(l, L(z, l)\tilde{\psi}(z, l)dldz.
\]
Market clearance condition in this sector is then given by

\[ L_S + b \mu_u L_Q = (1 - \gamma)I + N_H(\overline{c} + c_p + \mu_x c_x) + Mc_e, \]

where \( L_S \) and \( L_Q \) are the size of the workforce in the two sectors, and \( \mu_u \) is the unemployment rate within the \( Q \)-sector.

c) **Labor Market Clearing:** With a normalized measure of workers, the size of the workforce in the \( Q \)-sector is \( L_Q = 1 - L_S \). Total production employment in the differentiated good sector is given by

\[ E_Q = N_H \overline{I} = N_H \int_z \int_l l\psi(z,l)dl dz = (1 - \mu_u)L_Q, \]

where

\[ \overline{I} = \int_z \int_l l\psi(z,l)dl dz \]  \hspace{1cm} \text{(33)}

is the average employment in differentiated goods sector. The measure of unemployed workers is then

\[ L_u = L_Q - E_Q = \mu_u L_Q. \]

The equilibrium condition for the labor market in the \( Q \)-sector requires that flows out of employment equal the flows into employment. Every period, a fraction \( \mu_l \) of workers in that sector are laid off due to exits and downsizing

\[ \mu_l = \frac{\int_z \int_l [1 - \mathcal{I}(z,l)] l\psi(z,l)dl dz + \int_z \int_l \mathcal{I}(z,l) \mathcal{I}^f(z,l) [l - L(z,l)] \psi(z,l)dl dz}{\int_z \int_l l\psi(z,l)dl dz} \]

Then, the equilibrium flow condition is

\[ \mu_u L_Q \overline{\phi} = (1 - \mu_u)L_Q \mu_l, \]

which yields the usual Beveridge curve

\[ \mu_u = \frac{\mu_l}{\mu_l + \overline{\phi}}. \]

On vacancies side, the aggregate number of vacancies in this economy is given by

\[ V = N_H \overline{v} = N_H \int_z \int_l v(z,l) \mathcal{I}^h(z,l) \frac{\tilde{\psi}(z,l)}{\mu_h} dl dz, \]
where
\[
\overline{\nu} = N_H \int_z \int_l v(z, l) \mathcal{I}^h(z, l) \frac{\tilde{\psi}(z, l)}{\mu_h} dldz,
\] (34)
is the average level of vacancies, and \(\mu_h\) is the fraction of hiring firms:
\[
\mu_h = \int_z \int_l \mathcal{I}^h(z, l) \tilde{\psi}(z, l) dldz.
\]
The total number of vacancies, \(V\), together with \(L_u = \mu_u L_Q\), determines matching probabilities \(\phi(V, L_u)\) and \(\tilde{\phi}(V, L_u)\) that firms and workers take as given.

d) **Firm turnover:** In equilibrium, there is a positive mass of entry \(M\) every period so that the free entry condition \((16)\) holds with equality. The fraction of firms exiting is implied by the steady state distribution and the exit policy function,
\[
\mu_{exit} = \int_z \int_l [1 - \mathcal{I}^x(z, l)] \psi(z, l) dldz,
\]
and measure of exits equals that of entrants,
\[
M = \mu_{exit} N_H.
\]

e) **Trade Balance:** Given the exact price index for imported goods,
\[
P_F = \tau_m \tau_c k \left[ \int_0^{N_F} p_F(n)^{1-\sigma} dn \right]^{1/(1-\sigma)},
\]
total domestic spending on imported varieties is given by
\[
E_F = \tau_m \tau_c k \int_0^{N_F} p_F(n) q_F(n) dn = D_H [\tau_m \tau_c k]^{1-\sigma},
\]
and domestic demand for foreign currency (expressed in domestic currency) is
\[
\frac{E_F}{\tau_m} = \frac{D_H [\tau_m \tau_c k]^{1-\sigma}}{\tau_m} = D_H \tau_m^{-\sigma} [\tau_c k]^{1-\sigma}.
\]
Tariff revenues collected by the home country government amount to \(T = \frac{E_F}{\tau_m} (\tau_m - 1)\). We assume all tariff revenues are returned to worker/consumers in the form of lump-sum transfers. Total export revenues are
\[
S_F = N_H \int_z \int_l s_F(z, l, \eta^o) \mathcal{I}^x(z, l) \psi(z, l) dldz,
\]
and since service goods are non-traded, balanced trade obtains when \( \frac{E_F}{r_m} = S_F \).
The exchange rate \( k \) moves to ensure that this condition holds. Balanced trade ensures that national income matches national expenditure.

6. Workers are indifferent between taking a certain job in the undifferentiated sector and searching a job in industrial sector: \( J^o = J^s = J^u \).
Appendix 3: Estimating the Revenue Function and Productivity Process

The Revenue Function

The equation we wish to estimate is:

\[
\ln R_{it} = \rho \ln R_{it-1} + (d_H + \mathcal{I}_it^x d_F) - \rho (d_H + \mathcal{I}_{it-1}^x d_F)
\]

\[
+ \alpha \left( \frac{\sigma - 1}{\sigma} \right) \ln l_{it} - \alpha \rho \left( \frac{\sigma - 1}{\sigma} \right) \ln l_{it-1} + \left( \frac{\sigma - 1}{\sigma} \right) \epsilon_{it} .
\]

But selection bias and simultaneity bias prevent us from consistently estimating this expression with ordinary least squares. The former problem occurs because firms choose whether to shut down partly on the basis of their \( \epsilon_{it} \) realizations, and the latter problem occurs because firms’ current exporting decisions (\( I_{it}^{x} \)) and employment levels (\( l_{it} \)) depend upon their current productivity levels.

Selection Bias and Identification

To deal with these problems, let \( I_{it}^{c} \) be an indicator variable that takes a value of 1 if the \( i \)th firm continues to operate in period \( t \), and 0 otherwise. Then, defining \( \xi_{it} = \epsilon_{it} - E [ \epsilon_{it} | I_{it}^{c} = 1, \ln R_{it-1}, \ln l_{it-1}, I_{it-1}^{c} ] \), the revenue function can be re-formulated as:

\[
\ln R_{it} = \rho \ln R_{it-1} + d_H (1 - \rho) + d_F (I_{it}^{x} - \rho I_{it-1}^{x}) + \alpha \frac{\sigma - 1}{\sigma} \ln l_{it} - \alpha \rho \frac{\sigma - 1}{\sigma} \ln l_{it-1} + \frac{\sigma - 1}{\sigma} \xi_{it},
\]

where the error term \( \xi_{it} \) has zero mean and is orthogonal to \( \ln R_{it-1}, \ln l_{it-1}, I_{it-1}^{c} \), and \( E [ \epsilon_{it} | I_{it}^{c} = 1, ... ] \). Also, although it is correlated with current exporting decisions, \( \xi_{it} \) is orthogonal to \( E [ I_{it}^{x} | I_{it}^{c} = 1, \ln R_{it-1}, \ln l_{it-1}, I_{it-1}^{c} ] \). These implications of our model can be used as the basis for a generalized method of moments (GMM) estimator that identifies the parameters of equation (A3.1). And the efficiency of this estimator can be improved by exploiting the moment condition \( E [ I_{it}^{x} (1 - e^{-d_F}) - x_{it} ] = 0 \), where \( I_{it}^{x} (1 - e^{-d_F}) \) is the share of exports in total sales implied by our model and \( x_{it} \) is the observed ratio of export revenues to total sales, which we treat as a noisy measure of true export intensity.

Identification further requires that these conditional expectations be non-linear functions of their arguments and/or that they condition on additional arguments that do not appear in equation (A3.2). Note that the dependence of \( \ln l_{it} \) on \( \epsilon_{it} \) does not prevent us from obtaining consistent estimates of these parameters because the coefficient on \( \ln l_{it} \) can be inferred from the coefficients on \( \ln l_{it-1} \) and \( \ln R_{it-1} \).
This estimation strategy requires that we calculate \( E[\epsilon_{it}|T^c_{it} = 1, \ln R_{it-1}, \ln l_{it-1}, T^x_{it-1}] \). To this end, recall that there is a threshold productivity level above which all firms with beginning-of-period employment level \( \ell_{it-1} \) will continue operating. Denoting this threshold productivity level \( g^*(\ell_{it-1}) \), the continuation condition is \( \ln z_{it} = \rho \ln z_{it-1} + \epsilon_{it} > g^*(\ell_{it-1}) \). Or, since

\[
\ln z_{it-1} = \frac{\sigma}{\sigma - 1} \left[ \ln R_{it-1} - (d_H^i + T^x_{it-1} d_F^i) \right] - \alpha \ln l_{it-1}
\]

by equation 26, continuation occurs when

\[
\frac{\epsilon_{it}}{\sigma_{\epsilon}} > \frac{g^*(\ell_{it-1}) - \rho \ln z_{it-1}}{\sigma_{\epsilon}} \overset{\text{def}}{=} g(R_{it-1}, l_{it-1}, T^x_{it-1}),
\]

and the probability of continuation can be calculated as

\[
p^C_{it} = 1 - \Phi \left[ g(\ln R_{it-1}, \ln l_{it-1}, T^x_{it-1}) \right], \tag{A3.3}
\]

where \( \epsilon_{it} \sim N(0, \sigma_{\epsilon}^2) \) and \( \Phi() \) is the standard normal cumulative distribution. Treating \( g() \) as a flexible function of its arguments, it follows that \( p^C_{it} \) values can be imputed from estimates of the probit function (A3.3), and the conditional expectation of interest can be calculated using well-known properties of the normal distribution (e.g., Maddala, 1983):^26

\[
E[\epsilon_{it}|T^c_{it} = 1, \ln R_{it-1}, \ln l_{it-1}, T^x_{it-1}] = \sigma_{\epsilon} \cdot M_{it},
\]

\[
\text{var}[\epsilon_{it}|T^c_{it} = 1, \ln R_{it-1}, \ln l_{it-1}, T^x_{it-1}] = \sigma_{\epsilon}^2 \cdot (1 - M_{it} \left[ M_{it} - \Phi^{-1}(p^C_{it}) \right]),
\]

where \( M_{it} = \frac{\phi(\Phi^{-1}(p^C_{it}))}{p^C_{it}} \) is the relevant Mills ratio and \( \phi() = \Phi'(\cdot) \).

Our estimation strategy also requires that we calculate \( E[T^x_{it}|T^c_{it} = 1, \ln R_{it-1}, \ln l_{it-1}, T^x_{it-1}] \). For this, note that firms above some threshold productivity level will choose to export, given \( (l_{it-1}, z_{it-1}) \). Thus, once again exploiting the normality of \( \epsilon_{it} \), we can write

\[
E[T^x_{it}|T^c_{it} = 1, \ln R_{it-1}, \ln l_{it-1}, T^x_{it-1}] = p^Y_{it} = 1 - \Phi \left[ h(\ln s_{it-1}, \ln l_{it-1}, T^x_{it-1}) \right], \tag{A3.4}
\]

where \( p^Y_{it} \) is the probability that firm \( i \) exports in period \( t \) and \( h(R_{it-1}, l_{it-1}, T^x_{it-1}) \) is a flexible function of its arguments.\(^{27}\) Hence \( E[T^x_{it}|T^c_{it} = 1, ...] \) can be calculated by esti-

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^26When estimating this probit, we use a flexible (translog) functional form for \( g(r_{it-1}, l_{it-1}, I^x_{it-1}) \).

^27It is interesting that lagged exports help predict current exports here, even though we have assumed away sunk entry costs. The reason is that, by (26), lagged exports help to explain lagged productivity.
mating the probit (A3.4) and retrieving the imputed $p_{it}^X$ values. Clearly, identification here comes from the non-linear form of the probit function.  

The Moment Conditions  To summarize, our GMM estimator is based on the moment conditions:

$$
E[\xi_{it} \ln R_{it-1}] = 0, \ E[\xi_{it} \ln \ell_{it-1}] = 0, \ E[\xi_{it} M_{it}] = 0, \ E[\xi_{it} T^x_{it-1}] = 0, \\
E[\xi_{it} p_{it}^X] = 0, \ E[\xi_{it}] = 0, E[\nu_{it}^e] = 0, \ E[\nu_{it}^x] = 0.
$$

where:

$$
\xi_{it} = \frac{\sigma}{\sigma - 1} \left[ \ln R_{it} - d_H(1 - \rho) - d_F(T^x_{it} - \rho T^x_{it-1}) - \rho \ln R_{it-1} \right] + \alpha \rho \ln \ell_{it-1} - \alpha \ln \ell_{it} - \sigma_e M_{it}, \\
\nu_{it}^e = \xi_{it}^2 - \sigma_e^2 \cdot \left( 1 - M_{it} \left[ M_{it} - \Phi^{-1}(p_{it}) \right] \right), \\
\nu_{it}^x = T^x_{it} (1 - e^{-d_X}) - x_{it}.
$$

While $\alpha\left(\frac{\sigma - 1}{\sigma}\right)$, $\rho$, $\sigma_e^2$, $d_X$, and $d_H$ can be estimated using the approach sketched above, $\alpha$ and $\sigma$ are not separately identified. We therefore set $\sigma = 5$, a value typical of the literature, and generate estimates for the remaining parameters. (Refer to Table 1 in the text.) Our results proved not to be sensitive to the inclusion of time dummies in A1.1. Accordingly, since our theoretical model presumes that the macro environment is stable, we focus our attention on the case in which they are omitted. As noted in section 3.2, however, the results did prove to be sensitive to the way in which our labor measure is constructed and to as the instrument set.

\footnote{Olley and Pakes (1996) develop a related strategy that posits a deterministic linkage between productivity shocks and investment levels. This allows them to get away from functional form as a basis for identification, but it is not an available option in the present setting.}
Appendix 4: Numerical Solution Algorithm

We begin our solution algorithm with exogenous values for $\tau, \tau^m, D_F,$ and $r,$ thereby immediately determining $J^o = 1/r.$ To compute the value functions, we discretize the state space on a log scale using 500 grid points for employment and 50 grid points for productivity. We set the maximum firm size as 7,500 workers and numerically check that this is not restrictive. In steady state, a negligible fraction of firms reach this size. We then:

a) Formulate guesses for $c_f, D_H, w_f(z, l), \eta$ and $\phi.$ Calculate $w_h(z, l)$ using equation (24).

b) Given $D_H, w_f(z, l), \eta, \phi$ and $w_h(z, l)$ calculate the value function for the firm, $V(z, l)$, using equation (11) and find the associated decision rules for exit, hiring and exporting. Calculate the expected value of entry, $V_e$, using equation (16). Compare $V_e$ with $c_e.$ If $V_e > c_e,$ decrease $D_H$ (to make entry less valuable) and if $V_e < c,$ increase $D_H$ (to make entry more valuable). Go back to Step 1 with the updated value of $D_H$ and repeat until $D_H$ converges.

c) Given $w_f(z, l), \eta, \phi$ and the converged value of $D_H$ from Step 2, update $w_f(z, l).$ To do this, first calculate $J^e(z', l')$ using equations (20) and (23), and imposing the equilibrium condition $J^u = J^o.$ Given $J^e(z, l),$ update firing wage schedule using equation (25). Compare the updated firing wage schedule with the initial guess. If they are not close enough go back to Step 1 with the new firing wage schedule and repeat Steps 1 to 3 until $w_f$ converges. Note that if firing wages are too high, then $J^e(z, l),$ the value of being in a firm at the start of a period, is high, since the firm is less likely to fire workers. A high value of $J^e(z, l),$ however, lower firing wages. Similarly, if the firing wages are too low, then $J^e$ is low, which pushed firing wages up.

d) Given $\phi,$ the converged value of $D_H$ from Step 2, the converged value of $w_f(z, l)$ from Step 3, calculate the trade balance. In order to do this:

(a) Given firms decisions, calculate $\psi(z, l)$ and $\tilde{\psi}(z, l),$ the stationary probability distributions over $(z, l)$ at the end and interim states, respectively.

(b) Given $\tilde{\psi}(z, l),$ calculate average number of vacancies and the average employment in differentiated goods sector using equations (34) and (33).
(c) Take a guess for $N_H$. Given $N_H$ and $\underline{\pi}$, calculate the mass of unemployed $L_u$ in differentiated goods sector from

$$\phi(V, L_u) = \frac{M(V, L_u)}{V} = \frac{L_u}{((v N_H^\theta + L_u^\theta)^{1/\theta}},$$

which is one equation in one unknown. Given $N_H, \bar{\ell}$ and $L_u$, calculate the size of the workforce in the $Q$-sector is $L_Q$ from

$$N_H \bar{l} = L_Q - L_u.$$

Given $N_H, L_S = 1 - L_Q, M$ (mass of entrants), and $I$ (aggregate income), check if supply and demand is equal in the service sector

$$L_S + b\mu_n L_Q = (1 - \gamma) I + N_H(\underline{\pi} + c_p + \mu_x c_x) + Mc_e.$$

If the supply is greater than the demand, decrease $N_H$ and if supply is less than demand, increase $N_H$. Repeat until $N_H$ converges. Repeat Step 4c until $N_H$ converges.

(d) Given the value of $N_H$ from Step 4c, calculate exports and imports. If exports are larger than imports, lower $\eta$ and if exports are less than imports, increase $\eta$. Go back to Step 1 with the updated value of $\eta$, and repeat until convergence.

e) Given the converged value of $D_H$ from Step 2, the converged value of $w_f(z, l)$ from Step 3, and the converged value of $\eta$ from Step 4, update $\phi$. In order to do that, first calculate $EJ_h^e$ using (20). Then find $\tilde{\phi}$ from

$$\tilde{\phi} = (1 - \phi)^{1/\theta}.$$

Given $EJ_h^e$ and $\tilde{\phi}$, calculate $J^u$ using (18). If $J^o > J^u$, increase $\phi$ (to attract workers to differentiated goods sector) and if $J^o < J^u$, we lower $\phi$ (to make the differentiated goods sector less attractive). Go back to Step 2, and repeat until $\phi$ converges.

f) Calculate average wages in equilibrium. Check if $c_f$ is the right multiple of average wages. If not, update $c_f$ and go back to Step 1.
Estimation Code  The above algorithm solves the model for a given set of exogenous parameter values, including the cost of entry $c_e$. When we estimate the benchmark model to obtain parameter estimates, we: i) use the empirical value of $\eta$, ii) take the value of $D_H$ estimated in the first stage where we estimate revenue function parameters, iii) set $c_e$ such that free entry holds. This enables us to skip Step 2 and 4 in the calibration. When we do policy experiments by varying the parameters related to trade costs, the values of $D_H$ and $\eta$ change endogenously, so we use the complete algorithm to solve the model.
References


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