Scale and the Origins of Structural Change*

Francisco J. Buera† and Joseph P. Kaboski‡

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Abstract

Structural change involves a broad set of trends: (i) sectoral reallocations, (ii) rich movements of productive activities between home and market, and (iii) an increase in the scale of productive units. After extending these facts, we develop a model to explain them within a unified framework. The crucial distinction between manufacturing, services, and home production is the scale of the productive unit. Scale technologies give rise to industrialization and the marketization of previously home produced activities. The rise of mass consumption leads to an expansion of manufacturing, but a reversal of the marketization process for service industries. Finally, the later growth in the scale of services leads to a decline in industry and a rise in services.

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†Northwestern University, f-buera@northwestern.edu

‡Ohio State University, kaboski.1@osu.edu
The rise of large scale technologies is a central aspect of the process of development. Industrialization involves the implementation of technologies with high and rapidly growing productivity. These technologies are not only more productive but also involve a dramatic increase in the optimal scale of productive units. For example, in the United States, the workers per manufacturing firm increased seven fold between 1850 to 1950, from nine to sixty-three workers. During the same period, real income per capita had risen only five-fold.1

The emergence of scale technologies is a central theme in the modern literature on industrialization.2 This literature, in examining the potential obstacles to development stemming from scale technologies, has focused on the transition from traditional, small-scale, cottage industry technologies to large-scale manufacturing during development. The sole emphasis on the rise of the manufacturing sector ignores a broader set of changes taking place during the transition to a modern economy. Indeed, at its peak, industry constitutes less than half of an economy’s measured output. A broader view of the process of development, dating back to Kuznets (1973), includes the changes in the relative importance of broadly defined sectors, (e.g., agriculture, manufacturing and services), the marketization of home production, and the introduction of modern technologies into the household.

In this paper, we document facts on scale patterns, sectoral trends, and home vs. market decisions, and provide a unified theory for them. In our theory, scale technologies are the origin of structural change. That is, a model designed to be consistent with cross-sectoral and secular evidence on the scale of productive units delivers strong predictions for the movement of production between sectors and between home and the market. These predictions are consistent with the data. In particular, large-scale technologies drive production out of the home and into the market, but these patterns reverse when households begin directly purchasing productive intermediates en masse, and much modern service production moves back into the home. This drives an expansion of the manufacturing sector at the expense of agriculture and services. Finally, large-scale technologies may play a role in the more recent counter-reversal, in which home production has declined relative to market labor and services.

We model the role of scale in a dynamic economy with two technologies, a high-growth, modern technology and a stagnant, traditional, subsistence technology. The modern is a multi-stage technology with product units of three different scales: home production, services, and manufacturing. The optimal scales of these units arise because of specialized and indivisible manufactured (intermediate/capital) goods needed for modern production.

- Home production using the modern technology involves production only for (very customized) self-consumption. It is therefore the smallest scale

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2 See, for example, important contributions by Murphy, Shleifer, and Vishny (1989a, 1989b), Matsuyama (1992), and Banerjee and Newman (1993).
and most costly to produce, but its output is customized and therefore has utility benefits.

- **Services** production is a substitute for home production. It has an intermediate optimal scale, and its productive efficiencies may or may not outweigh its lower utility from limited customization.

- **Manufacturing** requires the largest fixed intermediates/capital and therefore operates at the largest scale. Given the large fixed costs and efficient scale in manufacturing, home production of manufactured goods is not a relevant alternative.

In contrast to the modern technologies, traditional production requires no intermediate/capital goods and offers no advantages of scale. Hence all production occurs in the home.

On the consumption side, agents hold a continuum of satiable wants ordered by the cost of producing output to satisfy them. Wants are symmetric. That is, although there is a difference in utility between home and market produced output, all wants are subject to this choice and so offer the same potential utility.

The model identifies two forces at work that determine the relative growth of the manufacturing, service, and home production over the development process. The first force ("marketization") leads to a relative increase in both industry and market services at the expense of the traditional home technology. The modern technology experiences technological progress, which the home is not able to fully replicate because of its necessary small-scale of operation. It is the large-scale of production of new technologies that draws production out of the home and into the market place. These modern scale technologies involve both industry and services, however, and this marketization force leads to an expansion of both, and a decline in traditional home production. This force is most important early in industrialization, when traditional home production is prevalent, and marketization is strongest for technologies with large optimal scales, high fixed intermediate/capital costs, and low utilization rates in the home (e.g. railroad production and rail travel).

The second force ("mass consumption") further drives the growth of manufacturing, but involves the choice to operate the modern technology at home rather than purchase market services. We call this force “mass consumption” because the choice of modern home production requires that consumers directly purchase manufacturing inputs, and generally requires more manufacturing inputs per unit of output (e.g. commuting separately in cars rather than riding together in a bus). Relative to modern market production of services, modern home production therefore yields less market production of services, but requires more manufactured goods production. Mass consumption often occurs later than marketization, as the costs of intermediates fall relative to income, but may occur immediately if disutility of market consumption is high, utilization is high, or the cost of intermediate goods are small (e.g., food and clothing services).
With sufficient heterogeneity in scale or the size of intermediate goods requirements, the model yields rich product cycles between the home and market. The production of goods is large scale and has a single transition from home to the market. Services for which scale is intermediate (e.g., local transportation, laundering) may move from the home (traditional) to the market, and back to the home (modern) over the course of development, while the smallest scale services may simply transition from traditional home to modern home production (e.g., cooking with a wood-burning stove to cooking with a gas/electric range).

Our emphasis on the link between scale technologies, sectoral allocations, and the home production margin is consistent with several additional empirical facts. First, the increase (and peak) in manufacturing’s share in value-added coincides with an increase (and peak) in the share of non-food goods in consumption, which is consistent with the mass consumption force. Second, the peak in manufacturing in the U.S. also corresponds to a peak in the amount of time spent in home production (Ramey and Francis, 2007). Market labor supply is known to be U-shaped over development (Goldin, 1994, Schultz, 1991), and the trough in labor supply in the U.S. also corresponds to the peak in home production. Finally, the post-1950 decline in industry relative to services is associated with growth in the scale of services, a decline in home production, and an increase in labor supply.

We conclude the introduction by reviewing related literature, in order to delineate our relative contribution, and underscore that our emphasis on scale and a broader view of structural change is important for policy and measurement. After this literature review, Section 2 develops the broad facts of structural change that motivate the paper. Section 3 introduces the model while Section 4 presents the main results. Section 5 discusses two extension of the basic model: (i) the case with heterogeneity in the scale of services; and (ii) a model with explicit durability of intermediate manufacturing inputs (capital). Section 6 compares some testable implications of our model with the data, and Section 7 concludes.

Related Literature Our empirical work extends and complements that of Kuznets (1957), Chenery and Syrquin, (1975), and Kravis et. al, (1984) by using updated sources and broadening the data in terms of number of countries and panel length. We use broadly comparable sources, and use our theory to make classification decisions for areas where discrepancies hold across methodologies.

Indeed, our theory of scale as a sectoral distinction gives guidance to empirical work, which has debated how to best classify handicrafts/cottage industries, which are particularly prevalent in less developed countries.3 Given their small-scale, we classify them as services. In our view, the scale of a productive unit

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3Dean and Cole (1967, pp. 138-139) describe the problems of classification that arose from the “radical transformation” of the structure of the British economy. Many occupations were classified in “retail trade and handcraft” in the 1831 census (e.g., wood and furniture, shipbuilding, printing, fur and leather, dressmaking, watches, toys and musical instruments, food/drink and also iron founders, weavers, dyers, and paper) were classified as manufacturing in later censuses.
reflects the extent of customization, with manufactured goods offering the least, home production offering the most, and services being intermediate. In this way, our paper is related to the view of Reid (1935) that manufacturing produces objects while services produce “circumstance” (location, condition, etc.), and Locay’s (1990) view that more customized activities are produced in the household.4

An existing theoretical literature (e.g., Murphy, Shleifer, and Vishny, 1989a, 1989b, Matsuyama, 1992, and Banerjee and Newman, 1993) has examined the role of scale in development, but in a different context. They showed how fixed costs involved with large scale technologies can lead to poverty traps in the presence of frictions, but focused on a single modern sector. In our broader view, where scale differs across sectors, frictions can have differential impacts across sectors as in Buera, Kaboski, and Shin (2007), Erosa and Hidalgo (2007), or Rajan and Zingales (1998).5 Differential effects on services and industry can be important since the sectors differ in their tradability and their contribution to investment.

Murphy, Shleifer, and Vishny (1989b) and Matsuyama (2002) also demonstrate the importance of mass consumer demand of manufactured goods in development. Katona (1964) emphasizes household investment in durables an important characteristic of “mass consumption”, however, and we draw a unique link between these durables and the growth of household production and manufacturing relative to services. Lagakos (2007) studies retailing and automobiles, a particular example of this link. Given the role of scale, frictions in consumer credit markets and the distribution of income may also play a role in industrialization.

Our model also underscores the measurement problems of home production and home labor, and so is related to recent work on the growth of the service sector and labor supply (Ngai and Pissarides, 2007, Rogerson, 2007). This approach diverges the recent literature on balanced growth and structural change (e.g., Acemoglu and Guerrieri, 2007, Ngai and Pissarides, 2007, and Kongsamut, Rebelo, and Xie, 2001, Foellmi and Zweimueller, 2006). Instead, we model a transition from stagnation to modern growth as in Galor (2005), Galor and Weil (2000), Gollin, Parente, and Rogerson (2007), and Hansen and Prescott (2005).

4Statistical agencies uses various categories to group industries according to their type of products. For example, according to the NIPA "goods are tangible products that can be stored or inventoried, services are products that cannot be stored and are consumed at the place and time of their purchase." Within goods, manufacturing establishments are defined, according to the NAICS, as those "engaged in the mechanical, physical, or chemical transformation of materials, substances, or components into new products." Such definitions are problematic practically (many classifications do not adhere to the definition) and conceptually (distinctions are not economically meaningful).

5The scale-dependent policies as emphasized by Guner, Ventura, and Yi (2006) can also affect sectors differentially.
1 Facts of Structural Change

“The rate of structural transformation of the economy is high. Major aspects of structural change include the shift away from agriculture to non-agriculture pursuits, and, recently, away from industry to services; a change of the scale of productive units, and a related shift from personal enterprise to impersonal organization of economic firms, with a corresponding change in the occupational status of labor.” (Kuznets, 1971, 2)

This section documents key facts on three aspects of structural change: growth in the scale of productive establishments, sectoral reallocations of production, and rich dynamics between home and market production.

1.1 Large Scale Technologies

In the model of the next section, scale is a proxy for the cost differential between home-scale and market-scale production. In the data, we will focus on workers per establishment as our preferred metric of scale in the market. Workers per enterprise is another alternative, but firms are often driven by contracting rather than technological considerations. The facts we present hold for either metric, however.

The first fact is that the growth process involves the introduction of large scale technologies in both industry and services. The industrial revolution involved the gradual spread of the factory system, characterized by the staggered arrival of a series of large-scale technologies (Mokyr, 2001, Scranton, 1997). Advances in agriculture in the early 18th century (e.g., seed drills, iron plows, threshing machines, and most importantly, enclosures) were a precursor, but the industrial revolution took off as technologies such as textile milling, iron production, mining, and steam power became increasingly economically viable. All led to increases in the scale of production, and required large capital investments. Similarly, the technologies of the second industrial revolution in the late 19th and early 20th century, such as steel, concrete, paper and chemicals, internal combustion engines, electricity, and food processing, led to even larger scales of efficient production, as did increased mechanization in agriculture (tractors, harvesters, etc.).

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6 In the model, output or capital would be related measures.
7 In some cases, however, the firm is the appropriate level that a technology is operated (e.g., Walmart and inventory management).
8 Textiles provide an example of the staggered arrival of technologies, which took over a century to fully move to large scale production. As Mokyr (2001) describes, cotton spinning, carding, bleaching, and printing were mechanized relatively early and moved to factory production, while weaving production remained in the home until the power looms arrival in the 1820s. Combed wool spinning was mechanized early, but the combing process itself was not mechanized until the mid-19th century. Hand production of worsted wool and linen lasted even longer.
The 19th century U.S. censuses of manufacturing made available by Atack and Bateman (1999) support the narrative history. Most manufacturers were still small-scale, with the median establishment employing just three workers in 1850, but the larger means indicate some larger scale producers in 1850. Scale grew in most industries between 1850 and 1870. The scales of industries associated with the new technologies (steel, textiles, paper, engines, farming machinery) were an order of magnitude larger, and had the largest increases in scale from 1850 to 1870. Appendix A presents these data from the 1850 and 1870 census of manufacturers for the major industries that can be compared over time.

Scale technologies were not particular only to goods production, but required services in their delivery (Chandler, 1990). Services, transportation, retail trade, and wholesale trade, in particular, were important elements even in early industrialization (Mokyr, 1990, Chandler, 1990). Canals, steam power, adding machines and cash registers, and other new office technologies led to an increase in the scale of services (Broadberry, 2006).

The second fact is that although modern services involved scale technologies, manufacturing technologies operate on a much larger scale. Even in the census of manufacturers, the smallest scale industries are those most commonly associated with services (dairy, bakeries, crop services, repair shops). For example, there was a large increase in the scale of meat products from 1850 to 1870 that may reflect a transformation of this industry from butchers to meat packers.11

The histograms of establishment size in Figure 1 show more generally that services are overwhelmingly small scale relative to industry. The vertical bar indicates the average scales of 47 and 14 for 4-digit (SIC) manufacturing and services, respectively. Despite the wide variance of scale in industry, the distributions overlap very little with most of the mass in services being below ten, and most of the mass of manufacturers being greater than ten. The difference is scale is true across each broad industry in the goods sector (including agriculture, mining, utilities, and manufacturing) and services sectors (transportation, services, public administration) with the exception of construction, which is typically in the industrial sector, but has many service-like characteristics.12 The identical patterns hold for enterprise size rather than establishment size with average scales of 57 and 18, respectively.

10 Atack (1985) provides further evidence for the U.S., while Sokoloff (1984) and Sicise (1994) provide evidence for early 19th century northeastern U.S. and 19th century France, respectively.

11 At times, scale has been used as an explicit basis for classification. For example, in the 1927 census, producers of confectionaries, ice cream and sheet iron were deemed to be manufacturers (as opposed to services) if annual production was at least $20,000.

12 For example, construction is non-tradable, and much of construction consists of small-scale contract work for which home production is a viable alternative.
1.2 Sectoral Reallocations

We extend Kuznets’ stylized development patterns for reallocations across industry, services and manufacturing with longer time series and a wider set of countries. Utilizing recent independent work by economic historians, we have assembled reliable extended time series of current price value-added share data for 30 countries, covering six continents and different levels of current development. (See documentation and data at http://faculty.wcas.northwestern.edu/~fjb913/BK2_DataAppendix.zip for details.)

The data series are summarized Figure ??, which shows value-added shares vs. real income per capita for industry (top panel), services (middle panel), and agriculture (bottom panel). Beyond the well-known decline in agriculture in the lower panel, two important features are discernible.

First, the share of manufacturing is hump shaped over development. Of the 30 countries, 21 – including all high income countries – have experienced an increase and then decline in industry, while the remaining lower income countries

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13 These countries include Argentina, Australia, Brazil, Canada, Chile, China, Colombia, Denmark, Egypt, France, Germany, India, Indonesia, Italy, Japan, Korea, Mexico, Netherlands, Norway, Pakistan/Bangladesh, South Africa, Spain, Sri Lanka, Sweden, Switzerland, United Kingdom, United States, Taiwan and Thailand. Based on Maddison (2005), our data covers: 68 percent of world population and 80 percent of world GDP in 2000; 70 percent and 74 percent, respectively, in 1950; and 40 percent and 60 percent, respectively in 1900. Although the numbers are lower for 1900, since the longer time series include Western Europe and its offshoots, we cover a much larger share of the population and economic activity undergoing large structural change at the time.
Figure 2: Sectoral Shares vs. Log Income per Capita for Country Panels
have only (yet) experienced the increase in industry. For these 21 countries, the peak share averages 0.40 (std. dev: 0.05) and occurs at an average per capita income of $7100 (st. dev.: $1800). Using this $7100 threshold to divide the country-year observations in the sample, regressions of industry’s share of country \( j \) on its log real income per capita \( \ln y_j \) that include country-specific fixed-effects \( (\alpha_j) \) yields the following results (standard errors in parentheses):

\[
< \$7100 \text{ sample: } \text{Ind. Share}_j = \alpha_j + 0.11 \ln y_j \quad (0.00)
\]

\[
\geq \$7100 \text{ sample: } \text{Ind. Share}_j = \alpha_j - 0.13 \ln y_j \quad (0.01)
\]

Second, services constitute a substantial share of output even early on, but exhibit a late acceleration with the decline of manufacturing. The 25 countries for which we have data at levels of per capita income below $2000 have services shares averaging 0.39 (std. dev: 0.07), which is comparable to the average share of agriculture in that income level, 0.40. The analogous split sample regression using service shares demonstrates the late acceleration:

\[
< \$7100 \text{ sample: } \text{Serv. Share}_j = \alpha_j + 0.07 \ln y_j \quad (0.01)
\]

\[
\geq \$7100 \text{ sample: } \text{Serv. Share}_j = \alpha_j + 0.20 \ln y_j \quad (0.01)
\]

1.3 Rich Dynamics Home vs. Market Movements

The location of particular productive activities changes markedly over development, and many activities exhibit rich product cycles. Historically, and even today in less developed economies, it has been difficult to construct truly meaningful national accounts, since they typically only encompass market activities. In these less developed economies, the advent and spread of industrialization involves the marketization of many formerly home-produced activities. As documented by Reid (1935), these included “spinning, weaving, sewing, tailoring, baking, butchering, soap-making, candle-making, brewing, preserving, laundering, dyeing, gardening, care of poultry,....child care, education, and the care of the sick” (p. 47).16

Two important industries that Reid omits are transportation and trade, both of which became much less home produced over time. Canals, railroads, and, later, mass transportation gradually replaced walking and horse-driven

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14 The UN National Accounts Main Aggregates Database, which includes sector specific numbers for a much larger cross-section of 161 countries but over a shorter time period (1970-2000), yields very similar results with low- and high-income sample coefficients of 0.07 and -0.12 for industry, respectively, and 0.04 and 0.18 for services.

15 Owner-occupied housing services and self-consumed agricultural output, particularly important in poorer, agrarian economies, are often imputed into national accounts, but home production of most other goods and services are not.

16 Reid’s observation was for the United States. Deane and Cole (1967) describe production in pre-industrial Britain, where market transactions were more prevalent, but small-scale production in the home still dominated. Even as industrialization increased market production of textiles, many productive activities were still contracted or "put out" to households.
transportation. Similarly, sale of home-produced output at markets became a smaller and smaller fraction of trade, as permanent retailers developed and distribution chains expanded.

Eventually, many of these marketized activities, as well as other market services, have moved back in the home. Buera and Kaboski (2006) show how many services declined in the twentieth century as important modern technologies and goods diffused to households. Important product cycles include the decline of transportation services, such as railroads, rail lines, and buses with the spread of the private automobile. The automobile was also related to the decline in neighborhood retail services (food, apparel, ice, fuel, dairy, “five and dime stores”), as was the spread of refrigerators and freezers.17 Similarly, the spread of washers, dryers, vacuums, microwaves, and other home appliances (see Greenwood et al, 2005) was accompanied by declines in domestic servants, launders, and dry cleaners. Francis and Ramey (2006) cite historical evidence that the spread of many household appliances were associated with increases in household production labor because activities (e.g., bread baking, laundry) moved from market to home production. Many newer activities that have started in the market have also moved toward home production. Examples include the relative decline of movie theaters (spread of televisions, VCRs, and DVD players), mail services (computers, fax machines), and recently internet cafes (computers, cable internet connections).

These examples of demarketization are quantitatively important. Together, Buera and Kaboski (2006) show that 75 percent of all declining service industries between 1950 to 2000 are associated with identifiable movements toward home production.

1.4 Summary

We have established 3 important facts:

**Fact 1** The industrial revolution involves large scale technologies, but the scale of manufacturing greatly exceeds that of services.

**Fact 2** Both modern industry and services play an important role early in development, but industry follows an extended hump-shape, while services exhibits a late acceleration.

**Fact 3** Economies experience rich product cycles between home and market production, including marketization and later demarketization of many services.

In the next section we present a model consistent with Fact 1, which in turn yields Facts 2 and 3.

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17 Lagakos (2006) examines the relationship between automobiles, retailing consolidation, and productivity in the context of developing countries.
2 A Theory of Structural Change

We model the consumption decision over a continuum of discrete wants. Individuals also choose whether to home produce or to procure these wants from the market. Production can be done using a traditional or a modern technology. Production using the modern technology requires the use of fixed amount of intermediate manufactured goods in combination with labor to produce up to a maximum scale. To satiate each want requires the use of both manufactured goods and services. In the model economy, as in the data, manufacturing differs from services by requiring a larger fixed cost and operating at a larger scale.

2.1 Preferences

There is a continuum of consumption wants indexed by \( z \). For each want \( z \), households make a discrete decision of whether to consume \( c(z) \) a service satisfying the want, and, if so, whether or not to home produce \( h(z) \) the service. Preferences over these decisions are represented by the following utility function:

\[
\tilde{u}(c, h) = \int_0^{+\infty} [h(z) + \gamma(1 - h(z))] c(z) \, dz
\]  

(1)

where \( h(z) \leq c(z) \in \{0, 1\} \). As will be clear with the discussion of technologies, \( z \) indexes the complexity associated with the production of a want.\(^{18}\)

Since \( \gamma \in (0, 1) \), home production yields more utility, perhaps because it avoids the disutility of public consumption (e.g., sitting next to others on the bus instead of driving one’s own car), or because it allows an individual to customize final consumption to his particular needs (e.g., driving one’s own car allows to use the preferred scheduled and route).\(^{19}\)

2.2 Technologies

Individual wants can be produced using a traditional or a modern (scale) technology. The traditional technology requires only labor as an input and experiences no productivity growth. The modern technology uses both labor and a fixed input of intermediate manufactured inputs to produce up to a maximum scale. Overtime, the productivity associated with the modern technology increases at a constant rate \( g \).

\(^{18}\)These preferences over a continuum of satiable wants are related to Matsuyama (2000, 2002) and Murphy, Shleifer and Vishny (1989). On the preference side, our innovation is to incorporate the home-production decision as in Kaboski and Buera (2007).

\(^{19}\)An alternative way to motivate home-production is to introduce transaction cost. See Buera and Kaboski (2006) from a discussion of the implication of this alternative model.
2.2.1 Traditional Technology

Individual wants can be produced using a traditional technology that requires only labor as an input and experiences no productivity growth:

\[ y_0(z) = z^{-1}l \]

Labor productivity declines with the index of wants \( z \), so that high \( z \) goods and services are more complex, and therefore more difficult to produce. The traditional technology does not require manufactured inputs, and therefore exhibits no scale economies. Therefore, all production using the traditional technology is done at home.

2.2.2 Modern (Scale) Technology

We also consider a modern production technology that requires a fixed input and is characterized by an efficient scale of production. In particular, production of goods and services associated with a want \( z \) requires a specialized intermediate manufactured input \( (k) \) of size \( q \). Given the intermediate input, the technology is linear in labor \( l \) up to a capacity of \( n \):

\[
y(z, t) = \begin{cases} 
0 & \text{if } k < q \\
\min \{ n, e^{\varphi z - \lambda t} \} & \text{if } k = q
\end{cases}
\]

Furthermore, \( \lambda < 1 \), i.e., the modern technology is relatively more productive than the traditional technology for more complex goods. The modern technology becomes relatively more attractive over time because technological change increases productivity at a rate \( g \), and consumption moves towards more complex wants.

Here \( n \) represents both the capacity and the efficient scale. For example, if a particular \( z \) were laundry, a service, then \( q \) might represent the cost of the laundry machine, which enables one to wash \( n \) loads of laundry when used at capacity.

At home, individuals will produce only one unit of output, and therefore underutilize purchased intermediates, i.e., produce at a higher cost scale. For this implication, it is important that the intermediates are indivisible (one cannot be half as productive with half a laundry machine) and specialized (a car cannot substitute for a laundry machine in doing laundry).

2.2.3 Distinguishing Sectors

The first distinction we make between sectors is to assume that goods production has a much larger efficient scale than services production. This is consistent with the evidence presented in Section 2.

As we show in the following section, production requiring large intermediate inputs \( q \) and/or done on a large scale \( n \) will tend to be performed on the market. For simplicity we model the extreme limiting case as \( q_M \to \infty \), so that manufactures are exclusively market produced. A further assumption of \( n_M \to \infty \),
and \( q_M/n_M \to 0 \) bounds the cost of goods. Thus, manufacturing production in the market simplifies to a constant return to scale technology:\(^{20}\)

\[
y_M(z, t) = e^{\gamma t} z^{-\lambda} I_M
\]

In what follows, to save on notation, we use \( q \) and \( n \) to refer to the intermediate input requirement and maximum scale associated with the production of services.

Secondly, we also make the further simplification that goods are only intermediates and not valued directly in the utility function. Goods will nevertheless be purchased as final consumption to be used in household production of services. Including goods as direct final consumption is feasible, but complicates the analysis without yielding much insight. Thus, for every \( z \) there is an intermediate good and a final service.\(^{21}\)

Finally, within the goods sector we distinguish agriculture as being the least complex goods, those below an arbitrary level \( z_A \).

The assumption that goods production is large scale makes it market rather than home produced.

### 2.3 Equilibrium

We can now state the household’s problem and the competitive equilibrium. For each want \( z \), the household makes three linked binary decisions: whether to consume or not, \( c(z) \), if so whether to home produce or not, \( h(z) \), and again if so, whether to use the modern technology in home production, \( m(z) \).

Normalizing labor as the numeraire, the household takes the wage and the prices of each good \( p_M(z) \) and service \( p_S(z) \) as given, and solves the following

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\(^{20}\) Alternatively, we can assume that \( q_M/n_M \to \alpha \), a constant that equals the intermediate goods’ share in manufacturing. In this case, manufacturing production in the market simplifies to a constant return to scale technology with fixed factor proportions:

\[
y_m(z, t) = e^{\gamma t - \lambda z} \min \left\{ (1 - \alpha) l_m, \alpha k_m \right\}.
\]

\(^{21}\) Furthermore, it can be argued that these two distinctions between goods and services are intimately related, as final consumption tends to be more customized and therefore less subject to large scale production (Locay, 1990).
static problem at each point in time:

$$\max_{m(z) \leq h(z) \leq c(z)} \int_0^{+\infty} [h(z) + \gamma (1 - h(z))] c(z) \, dz$$

s.t.

$$\int_0^{+\infty} c(z) \left[ h(z)m(z)q_{PM} (z) + \frac{[1 - h(z)] p_S (z,t)}{\text{service cons.}} \right] \, dz =$$

$$1 - \int_0^{+\infty} h(z) \left[ e^{-\lambda t} m(z)z^\lambda + \frac{[1 - m(z)] z}{\text{trad. home production}} \right] \, dz$$

(3)

The left-hand side of the budget constraint is total market expenditures, while the right-hand side is income/labor supply.

The first-order condition of whether to home produce or market purchase a particular service $z$ yields the central intuition for the model. A particular want is market produced iff:\textsuperscript{22}

$$\mu \left[ p_{PM} q (1 - \frac{1}{n}) \right] > 1 - \gamma$$

(4)

The bracketed term represents the cost-savings of market production. Both market and home production use labor (valued at the opportunity cost of time $w = 1$), but the market service requires paying only a fraction $(1/n)$ of the intermediate goods cost, as opposed to the full goods cost from purchasing the input. Households will use the market if the utility value of this cost-savings (left-hand side) exceeds the lost utility from consuming market- rather than home-produced output (right-hand side). Output that requires large or expensive intermediates (high $q$ or $p_{PM}$), or has a large efficient scale $n$ will be home produced. Hence, our assumptions that manufacturing requires large intermediates inputs $q$ and is done on a large scale $n$ justify the statement that manufacturing is market produced.\textsuperscript{23}

A competitive equilibrium is given by price functions $p_{PM} (z,t)$, $p_S (z,t)$, consumption, home production, and technology decisions $c(z,t)$, $h(z,t)$ and

\textsuperscript{22}The assumption that goods production is large scale makes it market rather than home produced. This could be seen clearly from the the household’s first-order condition of whether to home produce or market purchase a particular manufactured input, for the case $q_m$ and $n_m$ are finite,

$$\mu \left[ p_{PM} q (1 - \frac{1}{n}) \right] > 1 - \gamma.$$

As $q_m$ and $n_m$ go to infinity, the left-hand-side becomes arbitrarily large.

\textsuperscript{23}Strictly speaking, if manufactured goods are only intermediate goods there will not be a utility advantage associated with home-production of manufactures. The following heuristic argument should be understood within generalized model in which there is a utility gain associated with the home-production of manufactures, e.g., because of the possibility of customizing its design.
and $m(z,t)$ (associated with purchases of goods and services by households) such that:

i. given prices $p_M(z,t)$ and $p_S(z,t)$, $c(z,t)$, $h(z,t)$ and $m(z,t)$ solve (3);
ii. prices solve zero profits conditions, i.e.,

$$p_M(z,t) = e^{-gt} z^\lambda$$

and

$$p_S(z,t) = \left(1 + \frac{q}{p}\right) p_M(z,t);$$

iii. markets (i.e., for labor, each $z$ good, and each $z$ service) clear.

Next, we characterize the evolution of the structure of production of the economy. This process includes a shift from traditional technologies to modern (scale) technologies, changes in the wants that are home vs. market produced, and a transformation of the sectoral composition of output and employment.

3 Evolution of Structural Change

This section presents the results of the paper, which tie in closely with the facts presented in Section 2, given our assumption of large scale modern technologies and the larger relative scale of manufacturing. Proposition 1 describes the early transition from the pre-industrial to industrial scale economies and the marketization of previously home production activities, while Proposition 2 describes the later phase of industrialization in which activities return to the home as households begin mass consumption of modern technology intermediates. Thus, together the two propositions lead to rich product cycles, and a growth in manufacturing relative to services. Finally, Proposition 3 shows how the share of the service sector is increasing in its efficient scale of production.

3.1 Early Structural Transformation

For sufficiently low values of $t$, only the traditional technology is utilized. Since production using the traditional technology requires no specialized inputs, all production is done at home. Households consume the low $z$ goods first, since all $z$ are valued symmetrically, but the least complex output is cheapest to produce. An upper bound $z_0(t)$ defines the range of goods that are produced using the traditional technology. Early on, $z_0(t)$ also equals the most complex want that is satiated $\bar{z}(t)$. This upper bound remains fixed until industrialization.\footnote{This meshes with the historical evidence of the pre-industrial economy: relatively stagnant, with a very high fraction of production done at a small scale and at home (Reid, 1935, Deane and Cole, 1967, Mokyr, 1990, 2001).}

As productivity improves, the modern technology eventually becomes economically viable. The frontier $z(= z_0)$ is the first to be replaced by the modern
technology, but over time the modern technology becomes more productive for even the less complex output. During this period, the upper range of consumption $\mathcal{Z}(t)$ increases, and the upper range of consumption produced using the old technology $z_0(t)$ declines. In particular, there exists a point in time at which the modern technology overtakes the traditional technology for the most complex want that is satiated, $z = z_0$:

$$t_0 = \frac{1}{g} \left\{ \log \left( \frac{1 + \frac{q}{n}}{\gamma} \right) - \frac{1 - \lambda}{2} \log 2 \right\}$$

The timing of the onset of industrialization in the model depends positively on the share of intermediate specialized inputs in the modern technologies, $q/n$, and negatively on the rate of productivity growth in the modern technology and the disutility associated with market consumption.\textsuperscript{25}

The rise of scale technologies is associated with an increase in $\bar{z}(t)$ i.e., an expansion of the wants that are satiated, and a decrease in $z_0(t)$ (a decline of the range of wants satisfied through the traditional technology). Figure 3 illustrates this process. It describe the average cost per util as a function of the complexity of wants for the traditional (dotted) and modern (solid) technologies. Over time, the average cost per util for the modern technology declines.

Whether the new modern production that was previously traditional occurs as market or home production depends on the efficient scale of services relative to the utility advantage of home-production. If the scale of services is sufficiently small relative to the utility advantage of home-production, $1 + q/n > \gamma (1 + q)$, the advent of the modern technology is associated with a rise in the consumption of intermediate manufactured goods by households to be used as input in the home production of services. For these wants, services remain home produced, and there is just a transition from a traditional to a modern technology that utilizes intermediate inputs produced with a large scale technology. In the case of wants for which the scale of service production is large relative to the utility advantage of home-production, $1 + q/n < \gamma (1 + q)$, service production using the modern technology occurs on the market. In section 5, we generalize the model to allow for heterogeneity in the scale of services.

We summarize the previous discussion in the following proposition.

**Proposition 1 (Industrialization):** There exist two critical periods $t_0$ and $t_1$, $t_0 < t_1$, such that:

i) for $t < t_0$, only the traditional technology is utilized, the set of wants that are satiated remains fixed, and all production is done at home, i.e., $z_0(t) = \bar{z}(t) = \bar{z}(t) = 0$;

ii) for $t_0 \leq t < t_1$,

(a) the most complex wants are produced using the modern technology, $z_0(t) \leq z \leq \bar{z}(t)$, the set of satiated wants expands, $\partial \bar{z}(t) / \partial t > 0$, the set of wants produced using the traditional technology contracts, $\partial z_0(t) / \partial t < 0$; and

\textsuperscript{25}In modelling the onset of the industrial revolution as the moment in which a modern technology overcomes a traditional technology we follow Hansen and Prescott (2002). See also Stokey (2001).
Industrialization with “Marketization” of Services

\[ (1 + q) e^{-\sigma z^t} \]

\[ \frac{1}{\gamma} (1 + q/n) e^{-\sigma z^t} \]

Figure 3: Average Cost per Util as a Function of Complexity (z): Traditional Technology (red), Modern Market Technology (blue) and Modern Home Technology (pink)

(b) if \( \frac{1+q/n}{1+q} < (>) \gamma \), the most complex wants are satisfied in the market (at home) using the modern technology, and the service and industrial sectors (only the industrial sector) grow relative to agriculture.

3.2 The Rise of Mass-Consumption

Eventually the goods cost of producing any particular service \( z \) fall enough to induce direct household purchase of the market good and the home production of this service. Services begin returning to home production, but this time using the modern technology. This leads to the mass consumption of manufactured goods that are used in the production of services, and therefore is associated with sectoral reallocations in output: the return of market production to the home increases the demand for the given market good (by a factor of \( n \)), and decreases the purchase of the related service. Thus, the manufacturing sector experiences a boom relative to the service sector, and this contributes to the rising section of the hump-shaped manufacturing trend found in the data.

Proposition 2 (Mass Consumption): Assume \( (1 + q/n)/(1 + q) < \gamma \). Then, there exist \( t_1 > t_0 \) such that for \( t \geq t_1 \), the most complex home-produced wants are produced using the modern technology, \( z_0(t) < z < \bar{z}(t) < \tilde{z}(t) \), the set of wants satiated expands, \( \partial \tilde{z}(t)/\partial t > 0 \), the set of home-produced wants using the
modern technology expands, $\partial z(t)/\partial t > 0$ and $\partial z_0(t)/\partial t < 0$; and the industrial sector grows relative to the service sector.

Proposition 2 links a surge in the share of manufacturing to a surge in the share of manufactured goods in household consumption. We return to this testable implication in Section 4.

3.3 Large Scale Services and the Decline of Manufacturing

The previous sections have developed the model’s ability to deliver a long extended rise of industry. This section focuses on the model’s implications for the later decline in manufacturing, and corresponding rise in services.\textsuperscript{26}

The model predicts that the larger the scale of services, the larger the relative size of the services sector. There are two intuitive reasons for this result. First, the larger the scale, the less the goods cost per unit. That is, keeping $q$ constant, the share of intermediate goods is decreasing in scale. Second, the larger the scale, the larger the cost savings of market production of services (which produces at this efficient scale) relative to home production. The following proposition formalizes this.

**Proposition 3:** Both the share of market services (relative to market goods) and the ratio of market labor to home labor are increasing in the the scale of services, $n$.

In Section 4, we use the recent growth in the scale of services in the U.S. to test this implication of the theory.

4 Testable Implications

This section examines evidence on two testable implications of the theory: (1) evidence on the importance of the rise of mass consumption (Proposition 2), and (2) evidence on the link between the scale of market services and their share (Proposition 3).

4.1 Evidence on Mass Consumption

Proposition 2 predicts a link between the growth of household consumption of manufactured goods and the growing importance of the industrial sector relative to the service sector. It also links this consumption with the movement of activities into home production. Figure 4 shows that patterns in the value-added of industry and services are tied closely to household consumption of non-food

\textsuperscript{26} Buera and Kaboski (2006) focus on a related, and complementary explanation for the growth in services: their increasing skill intensity.
goods and services, respectively.\textsuperscript{27} Indeed, the peak of the share of the industrial sector coincides with peaks in the share of consumption expenditures on non-food goods. The peak in the fraction of non-leisure time spent in household production also coincides with these other two peaks. More generally, a U-shape in market labor supply over development is well-established and driven by the market hours of women (Goldin, 1994, and Schultz, 1991), which presumably reflects time reallocation between market and non-market production.

\subsection*{4.1.1 Scale and the Growth of Services}

The model also has relevance for the decline of industry and corresponding growth of services. Proposition 3 links the share of services to the optimal scale of market service production \( n \). Data from the County Business Patterns show a steady increase of 70 percent in the average scale of services from 1947-1997, while the scale in the goods sector has actually declined.\textsuperscript{28} Moreover, at a disaggregate level the growth in the service sector has been dominated by services whose scale has grown, and who are now among the largest scale

\textsuperscript{27}The consumption data is from Lebergott (1996) and NIPA, while the fraction of non-leisure time spent on household production are from Ramey and Francis for the population aged 18-65.

\textsuperscript{28}Scale is again defined as workers per establishment or workers per firm. In 1974, there is a change from a "reporting unit" (firm) concept to establishment. The pre- and post-1974 changes are 59 and 17 percent, respectively.
services. Using scale and payroll information by 3-digit level from the 1959 and 1997 County Business Patterns, OLS regressions yield the following estimates (with standard errors in parentheses):

\[
\Delta \text{share}_i = 0.20 + 0.69 \Delta \log \text{scale}_i
\]

where \( i \) represents 3-digit SIC industry (based on IPUMS 1950 coding, which allows us to link it to IPUMS data on schooling levels of workers in each industry), \( \Delta \text{share}_i \) is the absolute change in the percentage share of industry in total payroll payments between 1959 and 1997. The positive coefficient on \( \Delta \log \text{scale}_i \), the change in log employees per establishment, is significant at the one percent level. That is, industries that have grown in share have been the industries whose scale has increased.

This result is robust in two important ways. First, excluding the five largest and five smallest changes in shares still yields an estimate that is positive and still significant at a five percent level. Second, the relationship is not simply capturing the relationship between growth and skill intensity observed in Buera and Kaboski (2007). Controlling for \( \text{skill}_i \), the fraction of labor in an industry that was college-educated in 1940\(^{29}\), yields the following estimates:

\[
\Delta \text{share}_i = -0.31 + 0.71 \Delta \log \text{scale}_i + 5.01 \text{skill}_i
\]

The coefficient on \( \Delta \log \text{scale}_i \) is nearly identical and still significant at a one percent level. Thus, growth in scale appears to be independently related to the growth of disaggregate services.

4.2 Summary

We have presented theory and evidence of three phases of growth that include: (1) an early introduction of scale technologies leading to industrialization and a relative decline in the importance of agricultural output; (2) a somewhat later expansion of industry associated with mass consumption, and (3) still later expansion of services with the growth in their scale (which we develop in Section 4). Figure 5 illustrates these three phases of structural change in the model economy.

5 Extensions

5.1 Heterogeneity in the Scale of Services

So far, we have only considered two dimensions of heterogeneity: in the complexity of wants (\( z \)), and in the difference in fixed costs (\( q \)) and scale (\( n \)) of goods vs. services. Presumably, there is ample heterogeneity in the scale technology

---

\(^{29}\) Using the fraction that was college-educated in 2000 yields similar results for the role of scale, though the coefficient on skill is somewhat smaller given the higher education levels.
even within services. On one side of the spectrum, we find clothing services that requires relatively minor and divisible investments and, with the exception of specialized clothing items like tuxedos, are seldom provided in the market. On the other side are long distance travel services which require huge and lumpy investments and, at least initially, tend to be provided in the market. In this section we consider a simple extension of the basic model that incorporates this diversity.

As before, we assume a continuum of wants indexed by their complexity \( z \in [0, +\infty) \), each want requiring the production of manufactured inputs to be used with labor to produce the final service. In this extension, however, we allow for multiple wants of a given complexity \( z \) with different technologies to produce final services. To simplify the analysis, we consider two types of wants differing in the size of required specialized manufacturing inputs, i.e., \( i = 1, 2 \), with \( q_1 < q_2 \), \( 1 + q_1 < \frac{1}{\gamma} \left( 1 + \frac{\gamma}{n} \right) \) and \( 1 + q_2 < \frac{1}{\gamma} \left( 1 + \frac{\gamma}{n} \right) \).

In this simple extension, the evolution of the economy is divided in four stages characterized by three critical dates: \( t_{01}, t_{02} \) and \( t_1 \). In the first sub-period, all production is done using the traditional technology. This first stage, \( t \in (−\infty, t_{01}) \), mimics the traditional economy in the model described in the previous section with \( t_{01} = \frac{1}{g} \log \left[ 2^{\frac{1}{\gamma - 1}} (1 + q_1) \right] \).

\[30\] It is straightforward to generalize this model to the case of a continuum of wants of a given complexity \( z \), each of these wants indexed by the size of the fixed cost to provide the final service \( q \in [0, \infty) \). In this model, there would be effectively two types of wants: i) those that are industrialized without the marketization of services, \( q \leq \frac{(1/\gamma - 1)}{\gamma - 1(\gamma n)} \) (> 0 provided \( \gamma n < 1 \)); and ii) those that are (later) industrialized with the initial marketization of services, \( q > \frac{(1/\gamma - 1)}{\gamma - 1(\gamma n)} \).
The second stage, \( t \in [t_{01}, t_{02}) \), starts when it becomes profitable to use the modern technology for type 1 (small manufactured input requirement). In this stage, since \( 1 + q_1 < \frac{1}{\gamma} (1 + \frac{q_1}{n}) \), households directly purchase the intermediate manufactured inputs and home produce the final services themselves. Thus, this stage is characterized by a rise of manufacturing production and consumption relative to both agriculture/basic wants and services. The provision of clothing services is an example of such a want.

The third stage, \( t \in [t_{02}, t_1) \), is initiated when it becomes profitable to also use the modern technology for type 2 wants. Given their large manufactured input requirements, \( 1 + q_2 > \frac{1}{\gamma} (1 + \frac{q_2}{n}) \), type 2 services these are initially market produced. In this stage, both manufacturing and (market) services production and consumption rises relative to agriculture/basic wants. The provision of long distance transportation services with steam-engine locomotives are a clear example of these services.

The last stage is given by transition to home production of these type 2 services. This stage corresponds to the rise of mass consumption described in the model studied in the previous section, and is also characterized by rise in manufacturing production and consumption relative to both agriculture/basic wants and modern services.

This extension highlights another force leading to a rise of manufacturing production and consumption relative to both traditional sectors and modern services: the early modernization of wants that are characterized by relatively small scale technologies to provide the final service. As it was the case with the rise of mass consumption that we discuss in the previous section, the difference in the scale of production between home and market production of services, and among different market services, is at the center of the process of structural change.

5.2 Explicit Durability/Capital

In this section, we extend the basic model to allow for the durability of intermediate manufactured (capital) inputs. This is more in line with much of the earlier motivation which involves home durables and market capital goods. It also shows how the model maps into a more standard dynamic model that has similarities to the standard neoclassical growth model, but also yields insight into sectoral allocations.

In particular, we assume that each intermediate input faces one-hoss-shay depreciation at a constant hazard rate \( \delta \). As before, there is a continuum of wants indexed by \( z \) that are provided using labor and capital as inputs. For simplicity, we only consider the limit case where the modern technology is used in the production of all wants.

The preferences over the various wants within a period are still represented by the utility function (1), while the intertemporal preferences are represented the following time-separable utility function:
\[
\int_0^\infty e^{-\rho t} U (C(t)) \, dt
\]  
where \( C(t) = \int_{z^*}^{\infty} \left[ h(z,t) + \gamma (1 - h(z,t)) \right] c(z,t) \, dz \) and \( U(.) \) is a strictly increasing and concave function.

To simplify the exposition, we consider a decentralization in which households own the durable goods used in home production while the capital used by the market sector is owned by a competitive holding company. Under this assumption, the household’s problem simplifies to maximize (7) by choosing the stock of durable goods used in home-production \( z(t) \), the purchases of durable goods \( d(t) \), the most complex want that is purchase in the market \( \tilde{z}(t) \), and the stock of bonds \( B(t) \) subject to the time-t budget constraint

\[
\frac{\partial B(t)}{\partial t} + \int_{z(t)}^{\tilde{z}(t)} p_s(z,t) \, dz + d(t) = rB(t) + 1 - \int_{-\infty}^{\tilde{z}(t)} \frac{dz}{A(z,t)}
\]

where the left-hand side gives the purchases of new bonds, market services and durable goods and the right-hand side the capital and labor income; and the law of motion for the stock of durable goods

\[
\frac{\partial K^d(t)}{\partial t} = \delta K^d(t) + d(t)
\]

where \( K^d(t) = \int_{z^*}^{\infty} p_m(z,t) \, dz \), as all wants with complexity \( z < \tilde{z}(t) \) are home-produced and therefore \( q_s \) units of capital is required for production.

Standard optimal control arguments can be used to derive the dynamic system implied by the consumer’s problem. In what follows we describe a balanced growth path of this system.

Provided \( U(C) = -e^{-\sigma C} \), a balanced growth path exists and is characterized by the following two equations:

\[
r = \rho + \sigma g
\]

and

\[
p_m(\tilde{z},t) q_s (r + g + \delta) + \frac{1}{A(\tilde{z},t)} - p_s(\tilde{z},t) = (1 - \gamma) \frac{p_s(\tilde{z},t)}{\gamma}
\]

The first condition, the Euler equation, equates the interest rate to the rate of time preference plus a multiple of the growth rate, \( g \). The second condition equates the marginal cost of expanding the set of home-produced goods to the marginal return. The marginal cost (left-hand side) is given by the sum of the rental cost, \( p_m(\tilde{z},t) q_s (r + g + \delta) \) and the labor costs, \( \frac{1}{A(\tilde{z},t)} \), net of the savings associated with not having to satisfied this want in the market, \( p_s(\tilde{z},t) \). The marginal return (right-hand side) is proportional to the utility gain of home-production relative to market consumption of a given want, \( 1 - \gamma \). This last
condition determines the (constant) width of the set of services that are provided by the market, \( \bar{z}(t) - \bar{z}(t) \).

The model with durability allows us to study the effect of an increase in the cost of capital on the structural composition of consumption of this economy.

**Proposition:** The share of services in consumption \( c_s \) is a decreasing function of the cost of capital \( r \).

A larger cost of capital, due to a larger discount rate \( \rho \) or capital distortions, leads to a bigger cost advantage of market services that use more “efficiently” the capital input. Interestingly, this result is independent of whether services are more or less capital intensive than manufactures.\(^{31}\)

### 6 Conclusions

This paper has incorporated the efficient scale of productive units into theory of structural change. In particular, the introduction of large scale technologies, and the distinction between the scale of production in manufacturing, market services, and home produced services, help provide a unified explanation for broad trends of structural transformation, including not only scale, but also sectoral movements, and rich product cycles between home and market production.

We have also presented a potentially important explanatory factor in understanding the recent growth of the service economy: the increasing scale of services, and the increasing importance of large scale services. To the extent, that these large scale technologies may be improperly classified as services, these trends have implications for revisiting sectoral definitions in national income accounts.

Our emphasis on the importance of scale is relevant to the definition of the service sector in national accounting classification schemes. In particular, the NAICS system, which was instituted in the 1990s, moved in principle to a production method concept of industry. Still, it moved many large-scale information industries such as software publishing, printing, and motion pictures were classified into the service sector, while smaller scale activities such as bakeries and customized goods production were moved into manufacturing. Such classifications based on the content of what is produced rather than the production method lead to a less meaningful distinction between the sectors. Perhaps such classifications need to be revisited.

\(^{31}\)In this economy, the capital shares equals

\[
\alpha_m = \frac{q_m}{n_m} (r + \delta + g) \quad \text{and} \quad \alpha_s = \frac{q_s}{n_s} (r + \delta + g)
\]

\[
1 + \left( \frac{n_s}{n_m} - \frac{q_m}{q_s} \right) (r + \delta + g)
\]

for manufactures and for services. Thus, as long as \( r(t) = r \), we get constant factor shares. Furthermore, if \( \frac{n_s}{n_m} = \frac{q_m}{q_s} \), both sectors have the same capital intensity, \( \alpha_s = \alpha_m = \alpha \). Notice that constant factor shares across sectors are consistent with manufactures operating at a larger scale, i.e., both \( n_m \gg n_s \) and \( q_m \gg q_s \).
A Proof of the Results in the Paper

The various results in the paper follow from the characterization of the household’s problem. In this appendix we provide a characterization of this problem and we relate this characterization to the propositions in the paper.

The household chooses the set of wants to home produce using the traditional technology, \( z \in [0, z_0] \), the set of wants to home produce using the modern technology, \( z \in (z_0, z] \), and the set of want to market purchase, \( z \in [z, \bar{z}] \), where \( z_0 \leq z \leq \bar{z} \). Thus, households choose thresholds \( z_0, \underline{z} \) and \( \bar{z} \) to maximize

\[
\max_{0 \leq z_0 \leq \underline{z} \leq \bar{z}} (1 - \gamma) \underline{z} + \gamma \bar{z}
\]

subject to the budget constraint

\[
\int_{z_0}^{\underline{z}} q p_M (z, t) \, dz + \int_{\underline{z}}^{\bar{z}} p_S (z, t) \, dz = 1 - \int_0^{z_0} z \, dz - e^{-\gamma t} \int_{z_0}^{\underline{z}} z^\lambda \, dz
\]

where \( p_M (z, t) = e^{-\gamma t} z^\lambda \) and \( p_S (z, t) = (1 + \frac{\bar{z}}{\underline{z}}) p_M (z, t) \). The first-order conditions are

\[
\gamma + \theta_2 = \mu p_S (\bar{z}, t)
\]

\[
(1 - \gamma) + \theta_1 - \theta_2 = \mu \left( e^{-\gamma t} \bar{z}^\lambda + q p_M (\bar{z}, t) - p_S (\bar{z}, t) \right)
\]

and

\[
-\theta_1 = \mu \left( z_0 - e^{-\gamma t} \underline{z}^\lambda - q p_M (z_0, t) \right)
\]

where \( \mu \) is the Lagrange multiplier of the budget constraint, while \( \theta_1 \) and \( \theta_2 \) are the Lagrange multipliers of the inequality constraints, \( z_0 \leq \underline{z} \leq \bar{z} \).

There are four cases to be considered. The analysis of Cases 1-3 provides a proof of Proposition 2 (Industrialization), while Proposition 3 (Mass Consumption) is proven in the discussion of Case 4.

**Case 1**: \( z_0 = \underline{z} = \bar{z} \) (traditional economy) In this case, all production is done at home using the traditional technology. The most complex want that is satisfied using the traditional technology solves:

\[
\int_0^{z_0} z \, dz = 1
\]

or

\[
z_0 = 2^{\frac{1}{\lambda}}.
\]

This corresponds to the pre-industrial economy in which the set of wants that are satisfied remains constant over time. This will be the optimal solution as long as the following inequalities are satisfied

\[
\gamma \leq \mu p_S (\bar{z}, t),
\]

\[
(1 - \gamma) \geq \mu \left( e^{-\gamma t} \bar{z}^\lambda + q p_M (\bar{z}, t) - p_S (\bar{z}, t) \right).
\]
and

$$0 \geq \mu \left[ z_0 - e^{-gt} z_0^\lambda - qP_M (z_0, t) \right]$$

for $z_0 = \bar{z} = \bar{z} = 2^\frac{1}{2}$. Substituting in for $P_M (z_0, t) = e^{-gt} z_0^\lambda$, $P_S (z_0, t) = (1 + \frac{q}{n}) P_M (z_0, t)$, and $z_0 = \bar{z} = \bar{z} = 2^\frac{1}{2}$, and combining the three inequalities we obtain the following condition on $t$

$$2^\frac{1}{2} \leq \min \left\{ (1 + q) e^{-gt} 2^\frac{1}{2}, \left( 1 + \frac{q}{n} \right) e^{-gt} 2^\frac{1}{2} \right\}.$$  

That is, Case 1 holds for a sufficiently early date, i.e., $t < t_0$ with

$$t_0 = \frac{1}{g} \log \left( 2^\frac{1}{2} \min \left\{ (1 + q), \frac{1}{g} \left( 1 + \frac{q}{n} \right) \right\} \right).$$

**Case 2:** $z_0 = z < \bar{z}$ (Industrialization with marketization of services)

In this instance, the first-order conditions simplify to

$$\gamma = \mu \left[ 1 + \frac{q}{n} \right] e^{-gt} z^\lambda,$$  

$$1 - \gamma = \mu \left[ z_0 - \left( 1 + \frac{q}{n} \right) e^{-gt} z^\lambda \right]$$  

and

$$\left( 1 + \frac{q}{n} \right) e^{-gt} \int_{z_0}^{\bar{z}} z^\lambda dz = 1 - \int_0^{z_0} z dz.$$  

Combining (11) and (12) and integrating (13) yields two simple equations in $\bar{z}$ and $z_0$

$$\left( 1 + \frac{q}{n} \right) e^{-gt} \bar{z} = \frac{\gamma}{1 - \gamma} \left[ z_0 - \left( 1 + \frac{q}{n} \right) e^{-gt} z^\lambda \right]$$  

and

$$\frac{1}{\lambda + 1} \left( 1 + \frac{q}{n} \right) e^{-gt} \bar{z}^{\lambda+1} + \frac{z_0^2}{2} - \frac{1}{\lambda + 1} \left( 1 + \frac{q}{n} \right) e^{-gt} z_0^{\lambda+1} = 1$$  

Equations (14) and (15) define an upward and a downward sloping curve in the $(\bar{z}, z_0)$ space, respectively. It is straightforward to see that $\partial \bar{z} / \partial t > 0$ as both curves move upward with productivity. The effect of technological progress on the upper bound of the set of wants that are home produced using the traditional technology $z_0$ is given by

$$\frac{\partial z_0}{\partial t} = -\frac{g \gamma e^{gt}}{(1+q/n)} \left[ \frac{\lambda+1}{e^{\gamma (1+q/n)}} \left( 1 - \frac{z_0^2}{2} \right) + \frac{\lambda^\lambda+1}{z_0^{\lambda+1}} \right] - \frac{\lambda^\lambda+1}{z_0^{\lambda+1}} \left[ \lambda (1 - \frac{z_0^2}{2}) z_0^{\lambda} + \frac{1-\frac{\lambda^2}{2}}{(1+q/n)} e^{gt} + z_0^{\lambda+2} \right] < 0,$$

$$\lambda (1 - \gamma) \left[ -\frac{\lambda^\lambda+1}{e^{\gamma (1+q/n)}} \left( 1 - \frac{z_0^2}{2} \right) + \frac{\lambda^\lambda+1}{z_0^{\lambda+1}} + \frac{\lambda^\lambda+1}{z_0^{\lambda+1}} \right] < 0 \text{ (second order conditions)}.$$
Case 2 corresponds to the optimal solution if the following set of inequalities are satisfied:

\[
\begin{align*}
    z_0 &\leq z_0^0 e^{-qt} (1 + q) \text{ and } z_0 > z_0^\lambda e^{-qt} \min \left\{ \left( 1 + q \right), \left( 1 + \frac{q}{n} \right) \frac{1}{\gamma} \right\} \\
    \text{i.e., } z_0 &= z_0^{\min} \\
    \text{i.e., } z_0 < \max (\bar{z}, \bar{z}) \quad (16)
\end{align*}
\]

Together the conditions in (16) imply \( (1 + q) \frac{1}{\gamma} < (1 + q) \), the expression in Proposition 2.

Alternatively, these conditions can be expressed in terms of \( t \), where \( t_0 = \frac{1}{g} \log \left( 2^{\frac{1}{1+\gamma}} \frac{1}{1+q} (1 + \frac{2}{n}) \right) < t < t_1 \),

\[
t_1 = -\frac{1 - \lambda}{2g} \log \{ T \},
\]

and

\[
T = (1 + q/n) (1 + q) \frac{1}{\lambda + 1} \left[ \frac{1}{\lambda + 1} \left( \frac{\gamma}{1 - \gamma} \right)^{\frac{1}{\lambda + 1}} \left( \frac{1 + q}{1 + q/n} - 1 \right)^{\frac{1}{\lambda + 1}} \right] + \frac{1}{2} \frac{1 + q}{1 + q/n} - \frac{1}{\lambda + 1} \]

\[
> 0.
\]

Case 3: \( z_0 < z = \bar{z} \) (industrialization without marketization of services)

For this case, the first order conditions simplify to

\[
1 = \mu \left[ e^{-gt} \bar{z}^\lambda + q e^{-gt} \bar{z}^\lambda \right], \quad (17)
\]

\[
z_0 - e^{-gt} z_0^\lambda - q e^{-gt} z_0^\lambda = 0 \quad (18)
\]

and

\[
\frac{z_0^2}{2} + (1 + q) \frac{1}{\lambda + 1} \left[ e^{-gt} \bar{z}^{\lambda+1} - e^{-gt} z_0^{\lambda+1} \right] = 1 \quad (19)
\]

Using (18) we obtain a simple log-linear solution for the upper bound of the set of wants produced with the traditional technology \( \log z_0 = \frac{1}{1 - \lambda} \log (1 + q) - \frac{q}{1 - \lambda} t \). Clearly the upper bound on the set of wants that are consumed \( (\bar{z}) \) increases overtime.

This will be the solution provided \( (1 + \frac{q}{n}) \frac{1}{\gamma} > (1 + q) \) and \( t \geq t_0 \).

This completes the proof of Proposition 2. The discussion of Case 4 provides a proof of Proposition 3.

Case 4: \( z_0 < z < \bar{z} \) (rise of mass consumption)

This corresponds to the situation after the rise of mass consumption. In this case, the first order conditions simplify to

\[
\gamma = \mu \left( 1 + \frac{q}{n} \right) e^{-gt} \bar{z}^\lambda, \quad (20)
\]

\[
(1 - \gamma) = \mu \left[ e^{-gt} \bar{z}^\lambda + q e^{-gt} \bar{z}^\lambda \right] - \left( 1 + \frac{q}{n} \right) e^{-gt} \bar{z}^\lambda], \quad (21)
\]
\[ z_0 - (1 + q) e^{-gt} z_0^\lambda = 0, \]  
\begin{equation}
\frac{z_0^2}{2} + \frac{(1 + q)}{\lambda + 1} \left[ e^{-gt} z_0^{\lambda+1} - e^{-gt} z_0^{\lambda+1} \right] + \frac{(1 + \frac{q}{\lambda + 1})}{\lambda + 1} \left[ e^{-gt} z_0^{\lambda+1} - e^{-gt} z_0^{\lambda+1} \right] = 1.
\end{equation}

This corresponds to the optimal solution if the following set of inequalities are satisfied:

\[ \left( 1 + \frac{q}{n} \right) \frac{1}{\gamma} < 1 + q \text{ and } t > t_1. \]

Equation (22) can be solved for \( z_0 \)

\[ \log z_0 = \frac{1}{1 - \lambda} \log (1 + q) - \frac{g}{1 - \lambda} t \]

Using (20) and (21) we obtain a log-linear relationship between \( \bar{z} \) and \( z \)

\[ \log z = \frac{1}{\lambda} \log \left( \frac{(1 - \gamma) \left( 1 + \frac{q}{n} \right)}{\gamma q \left( 1 - \frac{1}{n} \right)} \right) + \log \bar{z} \]

Finally, using (23) it is straightforward to see that \( \bar{z} \) and \( z \) increase over time.

**Proof of Proposition 3:** In the long run, the share of services in output \( y_s \) equal:

\[ y_s = \frac{\int_{\bar{z}}^{\bar{z}} pS(z,t) \, dz - \int_{\bar{z}}^{\bar{z}} qM(z,t) \, dz}{\int_{\bar{z}}^{\bar{z}} qM(z,t) \, dz + \int_{\bar{z}}^{\bar{z}} pS(z,t) \, dz} \]

\[ = \frac{\int_{\bar{z}}^{\bar{z}} \bar{z}^\lambda \, dz}{q \int_{\bar{z}}^{\bar{z}} \bar{z}^\lambda \, dz + \left( 1 + \frac{q}{n} \right) \int_{\bar{z}}^{\bar{z}} \bar{z}^\lambda \, dz} \]

\[ = \frac{\bar{z}^{\lambda+1} - \bar{z}^{\lambda+1}}{(1 + \frac{q}{n}) \bar{z}^{\lambda+1} - (1 + \frac{q}{n} - q) \bar{z}^{\lambda+1}} \]

using that \( (\lambda + 1) \log \bar{z} = \frac{\lambda + 1}{\lambda} \log \left( \frac{(1 - \gamma)(1 + \frac{q}{n})}{\gamma q(1 - \frac{1}{n})} \right) + (\lambda + 1) \log \bar{z} \)

\[ y_s = \frac{1 - \left( \frac{(1 - \gamma)(1 + \frac{q}{n})}{\gamma q(1 - \frac{1}{n})} \right)^{\frac{\lambda + 1}{\lambda}}} {(1 + \frac{q}{n}) - (1 + \frac{q}{n} - q) \left( \frac{(1 - \gamma)(1 + \frac{q}{n})}{\gamma q(1 - \frac{1}{n})} \right)^{\frac{\lambda + 1}{\lambda}}} \]

Defining \( X = \left( \frac{(1 - \gamma)(1 + \frac{q}{n})}{\gamma q(1 - \frac{1}{n})} \right)^{\frac{\lambda + 1}{\lambda}} \), we get

29
\[ \frac{\partial y_s}{\partial n} = -\frac{q}{[(1 + \frac{2}{q}) (1 - X) + qX]^2} \frac{\partial X}{\partial n} + \frac{q (1 - X)}{[(1 + \frac{2}{q}) (1 - X) + qX]^2} > 0 \]

since \[ \frac{\partial X}{\partial n} = -\left( \frac{(1 - \gamma)}{\gamma} \right)^\frac{\lambda + 1}{\lambda} \left( \frac{1 + \frac{2}{q}}{1 - \frac{2}{q}} \right)^\frac{\lambda + 1}{\lambda - 1} \left[ \frac{q}{\gamma} \frac{1}{1 - \frac{2}{q}} + \frac{1}{\gamma} \frac{(1 + \frac{2}{q})}{1 - \frac{2}{q}} \right] < 0. \]

A.1 Explicit Durability

In the model with durable intermediate (capital) inputs, the household’s problem simplifies to

\[ \max_{B(t), \bar{z}(t), d(t), \bar{z}(t)} \int_0^\infty e^{-\rho t} U ((1 - \gamma) \bar{z}(t) + \gamma \bar{z}(t)) \, dt \]

s.t.

\[ \frac{\partial B(t)}{\partial t} + \int_{\bar{z}(t)}^{\bar{z}(t)} p_s(z, t) \, dz + d(t) = rB(t) + 1 - \int_{-\infty}^{\bar{z}(t)} \frac{dz}{A(z, t)} \]

and

\[ \frac{\partial K^d(t)}{\partial t} = \delta K^d(t) + d(t) \]

where \( K^d(t) = q_s \int_{-\infty}^{\bar{z}(t)} p_m(z, t) \, dz. \)

The Hamiltonian of this problem is given by

\[ H(t) = u((1 - \gamma) \bar{z} + \gamma \bar{z}) + \theta \left[ 1 + rB - \int_{A(z, t)}^{\bar{z}} \frac{dz}{A(z, t)} - \delta q_s \int_{-\infty}^{\bar{z}} p_m(z, t) \, dz - \int_{\bar{z}}^{\bar{z}} p_s(z, t) \, dz - d \right] + \mu \frac{1}{q_s p_m(\bar{z}, t)} \]

The Principle of the Maximum implies

\[ u'(C) \gamma = \theta p_s(\bar{z}, t) \quad (24) \]

\[ -\dot{\theta} = (r - \rho) \theta \quad (25) \]

\[ -\mu = -\rho \mu + u'(C) (1 - \gamma) \]

\[ -\theta \left[ \frac{1}{A(\bar{z}, t)} + \delta q_s p_m(\bar{z}, t) - p_s(\bar{z}, t) \right] - \mu \frac{1}{q_s p_m(\bar{z}, t)^2} \frac{\partial p_m(\bar{z}, t)}{\partial \bar{z}} \]

and

30
Performing standard manipulations we obtain the Euler equation

\[ \sigma \left[ (1 - \gamma) \frac{\partial \bar{z}(t)}{\partial t} + \gamma \frac{\partial \bar{z}(t)}{\partial t} \right] = r - \rho - \frac{1}{p_s(\bar{z}, t)} \frac{\partial p_s(\bar{z}, t)}{\partial \bar{z}} \frac{\partial \bar{z}}{\partial t} + g \]

and an equation for the equality of marginal cost of durables to the marginal return of durables

\[ q_s p_m(\bar{z}, t) (r + g + \delta) + \frac{1}{A(\bar{z}, t)} - p_s(\bar{z}, t) = \frac{1 - \gamma}{\gamma} p_s(\bar{z}, t) \]

**Holding Company’s Problem**

We assume that there is a competitive holding company that owns the capital stock used by the market sector. In particular, the holding company purchases manufacturing goods and rents these for a rental price \( R(z, t) \) to maximize the present value of profits

\[ \int_0^\infty \int_{-\infty}^\infty e^{-rt} [R(z, t) k(z, t) - I(z, t) p_m(z, t)] \, dz \, dt \]

subject to the law of motion for each type of capital

\[ \dot{k}(z, t) = I(z, t) - \delta k(z, t) \]

The firm’s problem solves the following Hamiltonian problem,

\[ H(t) = \int_{-\infty}^\infty \left\{ [R(z, t) k(z, t) - I(z, t) p_m(z, t)] + \kappa(z, t) [I(z, t) - \delta k(z, t)] \right\} \, dz \]

Necessary conditions are:

\[ \kappa(z, t) = p_m(z, t) \]

\[ -\dot{\kappa}(z, t) = R(z, t) - r\kappa(z, t) - \kappa(z, t) \delta \]

implying

\[ R(z, t) = p_m(z, t) \left( r + \delta - \frac{1}{p_m(z, t)} \frac{\partial p_m(z, t)}{\partial t} \right) \]

**Producer’s Problem**
Competitive firms produce market services and manufacturing goods. Zero profits imply

\[ p_m(z,t) = \frac{1}{A(z,t)} + \frac{q_m}{n_m} R(z,t) \]

and

\[ p_s(z,t) = \frac{1}{A(z,t)} + \frac{q_s}{n_s} R(z,t). \]

Using (30), we get

\[ p_m(z,t) = \frac{1}{A(z,t)} + \frac{q_m}{n_m} p_m(z,t) \left( r + \delta - \frac{1}{p_m(z,t)} \frac{\partial p_m(z,t)}{\partial t} \right). \]

Guessing \( \frac{1}{p_m(z,t)} \frac{\partial p_m(z,t)}{\partial t} = -g \) and using \( A(z,t) = e^{gt-\lambda z} \),

\[ p_m(z,t) = \frac{e^{\lambda z-gt}}{1 - \frac{q_m}{n_m} (r + \delta + g)} \] (31)

and

\[ p_s(z,t) = \frac{1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + \delta + g)}{1 - \frac{q_m}{n_m} (r + \delta + g)} e^{\lambda z-gt} \] (32)

where a bounded price of manufactured goods requires \( 1 - \frac{q_m}{n_m} (r + \delta + g) > 0 \). In this economy capital shares equal

\[ \alpha_m = \frac{q_m}{n_m} (r + \delta + g) \]

and

\[ \alpha_s = \frac{\frac{q_s}{n_s} (r + \delta + g)}{1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + \delta + g)} \]

Thus, as long as \( r(t) = r \) we get constant factor shares. Furthermore, if \( \frac{q_m}{n_s} = \frac{q_m}{n_m} \), \( \alpha_s = \alpha_m = \alpha \).

**Balanced Growth Path**

For a balanced growth path we need to have \( \ddot{z}(t) = \dot{z}(0) + \frac{q}{\lambda} t \) and \( \ddot{z}(t) = \dot{z}(0) + \frac{q}{\lambda} t \). Substituting these conditions into (28)

\[ \sigma \left[ (1 - \gamma) \frac{g}{\lambda} + \gamma \frac{g}{\lambda} \right] = r - \rho - g + g \]

or

\[ r = \rho + \sigma \frac{g}{\lambda} \]
From (29)

\[ q_s p_m (\bar{z}, t) (r + g + \delta) + \frac{1}{A(\bar{z}, t)} p_s (\bar{z}, t) = \frac{1 - \gamma}{\gamma} p_s (\bar{z}, t) \]

or

\[ q_s (r + g + \delta) + \frac{1}{p_m (\bar{z}, t) A(\bar{z}, t)} p_s (\bar{z}, t) = \frac{1 - \gamma}{\gamma} p_s (\bar{z}, t) \]

Using (31) and (32),

\[
q_s \left( 1 - \frac{1}{n_s} \right) (r + g + \delta) = 1 - \gamma \left[ 1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + \delta + g) \right] e^{\lambda(\bar{z}(0) - \bar{z}(0))}
\]

or

\[
q_s \left( 1 - \frac{1}{n_s} \right) = 1 - \gamma \left[ \frac{1}{(r + g + \delta)} + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) \right] e^{\lambda(\bar{z}(0) - \bar{z}(0))}
\]

The share of services in consumption equals

\[
c_s = \frac{\int_{\bar{z}}^{\infty} p_s (z, t) \, dz}{\delta q_s \int_{-\infty}^{\bar{z}} p_m (z, t) \, dz + q_s q_m \int_{-\infty}^{\bar{z}} p_m (z, t) \, dz + \int_{\bar{z}}^{\infty} p_s (z, t) \, dz}
\]

or

\[
c_s = \frac{\left[ 1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + \delta + g) \right] \left( e^{\lambda(\bar{z}(0) - \bar{z}(0))} - 1 \right)}{q_s (\delta + \frac{g}{\lambda}) + \left[ 1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + \delta + g) \right] \left( e^{\lambda(\bar{z}(0) - \bar{z}(0))} - 1 \right)} \tag{33}
\]

**Proof of Proposition:** Differentiating (33) with respect to the cost of capital we obtain

\[
\frac{\partial c_s}{\partial r} = \frac{q_s (\delta + \frac{g}{\lambda})}{q_s (\delta + \frac{g}{\lambda} + A)^2} \frac{\partial A}{\partial r}
\]

where

\[
A = \left[ 1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + \delta + g) \right] \left( e^{\lambda(\bar{z}(0) - \bar{z}(0))} - 1 \right)
\]

and

\[
\frac{\partial A}{\partial r} = \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + g + \delta) \frac{q_s (1 - \frac{1}{n_s})}{1 - \frac{1}{\gamma}} \left[ 1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + g + \delta) \right] - \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) \]

\[
\frac{1 - \frac{1}{n_s}}{1 - \frac{1}{\gamma}} \left[ 1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + g + \delta) \right]
\]
If $\frac{q_x}{n_x} - \frac{q_m}{n_m} > 0$, then $\frac{\partial A}{\partial r} > 0$ as the first two terms on the right-hand-side are equal to $\left(\frac{q_x}{n_x} - \frac{q_m}{n_m}\right) \left[e^{\lambda(z(t)-z(0))} - 1\right]$ and are therefore positive. In the case $\frac{q_x}{n_x} - \frac{q_m}{n_m} < 0$, we know that the sum of the first and third terms are positive as 

$$\left(\frac{q_x}{n_x} - \frac{q_m}{n_m}\right) (r + g + \delta) + 1 > 0.$$
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