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A THEORY OF AN INTERMEDIARY**

Yaron Leitner
Federal Reserve Bank of Philadelphia

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Inducing Agents to Report Hidden Trades: A Theory of an Intermediary

Yaron Leitner*
Federal Reserve Bank of Philadelphia

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Abstract

When contracts are unobserved, agents may have the incentive to promise the same asset to multiple counterparties and subsequently default. I construct an optimal mechanism that induces agents to reveal all their trades voluntarily. The mechanism allows agents to report every contract they enter, and it makes public the names of agents who have reached some prespecified position limit. In some cases, an agent's position limit must be higher than the number of contracts he enters in equilibrium. The mechanism has some features of a clearinghouse.

JEL codes: D82, G20

Keywords: mechanism design, optimal regulation, nonexclusive contracts, an intermediary, moral hazard, collateral, clearinghouse

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1 Introduction

Consider a bank that buys a credit default swap from AIG. The bank knows that AIG has enough capital to honor the swap agreement if the bank is its only client. But AIG might sell credit default swaps to many banks and create a liability that it cannot honor should it need to make a payment. This suggests a potential role for a central mechanism that monitors agents' positions. Indeed, the recent and ongoing financial crisis has led banks and regulators to work toward the establishment of a clearinghouse for credit default swaps.

If the central mechanism (e.g., a clearinghouse or a regulator) could observe all the contracts that agents enter, it could simply set position limits to make sure that agents do not enter into too many liabilities relative to their capital. But monitoring every contract that an agent can enter may be too costly, as agents may try to hide their transactions or make them too complicated to understand. I show that it is enough that the mechanism monitors only the contracts that agents *choose* to report. Thus, the mechanism provides a very cost-effective way to monitor agents' positions, beyond the fact that it saves on the cost of duplicate monitoring by individual agents.

The mechanism in this paper is an entity that keeps track of reported trades and makes some of the information it collects public. I focus on two types of mechanisms that differ in the amount of information they make public. The first is a *position limit mechanism*, which sets a position limit for every agent and makes public the names of agents who have reached their limit. In equilibrium an agent can enter contracts as long as he has not reached his limit, so the mechanism essentially sets an upper bound on the number of contracts that each agent can enter. The second mechanism is a *bulletin board mechanism*, which makes public all the information it has. Under both types of mechanisms, it is assumed that reporting is costly, so an agent reports only if he strictly prefers to do so.

The main result is that a position-limit mechanism can implement the same outcome that would be obtained if agents could not enter contracts secretly. In contrast, a bulletin board mechanism cannot always implement that outcome. I also show that in some cases the optimal position limit for an individual agent must be nonbinding in equilibrium; that is, the position limit must be higher than the number of contracts the agent enters in equilibrium.

The intuition is as follows: If an agent does not report a contract, he saves the reporting fee but allows his counterparty to enter more contracts than the position limit. This increases the counterparty's gain from strategic default, where he enters as many contracts as he can, planning to default on all. When the position limit is too high, the counterparty will always default whether the agent reports him or not. In this case, the agents will not enter a contract to begin with. When the position limit is too low, the counterparty will never default. In this case, an agent will not report to save the reporting fee. Thus, to induce reporting, the position limit cannot be too high and it cannot be too low. In some cases "not too low" means that the position limit must be higher than the number of contracts the counterparty enters in equilibrium.

The intuition above also explains why in some cases a bulletin board mechanism cannot achieve the second best. Suppose, for example, that an agent has one dollar, but the contractual payment is more than half a dollar. Clearly, the agent cannot deliver on a second (identical) contract, so once it becomes known that he has entered the first contract, no agent will enter a second contract with him. But this is like imposing a position limit of one, which might be too low to induce reporting.

An alternative to the mechanism is that agents put up cash as collateral. However, using collateral has an opportunity cost. Collateral reduces the incentive to default because (i) an agent who defaults loses the collateral and (ii) an agent may not have enough collateral to enter the number of contracts needed to make default profitable. The amount of collateral necessary to prevent default is less than the contractual payment. The optimal mechanism

may also require collateral but less than the amount needed without the mechanism.

Empirical predictions. According to the model, the gain from the optimal mechanism increases when the fixed cost per trade falls and/or the probability of finding a trading counterparty rises—both are features of a more liquid market.¹ The model also provides closed-form solutions and some comparative statics for the optimal amount of collateral (with and without the mechanism), the amount of investment, and the optimal position limits.

While the paper does not attempt to model any particular intermediary, the position limit mechanism, which is optimal in my setting, has some features of a clearinghouse. The clearinghouse may be part of a futures exchange or a stand-alone institution; it can clear exchange-traded contracts as well as over-the-counter products, such as swaps.² Clearinghouses around the world deploy a number of safeguards to protect their members and customers against the consequences of default by a clearinghouse participant. In addition to requiring collateral, the clearinghouse monitors the financial status of its members. On a daily basis (or sometimes more than once a day), the clearinghouse monitors and controls the positions of its members; periodically, the clearinghouse monitors financial statements, internal controls, and other indicators of financial strength; some clearinghouses (e.g., in Sydney and Hong Kong) also set capital-based position limits.³ These safeguards, which reduce the amount of collateral that clearinghouse members need to post, are more effective when clearinghouse members do not enter contracts secretly.⁴ In practice, the incentive to

¹This seems consistent with the observation that the London Clearing House started clearing over-the-counter interest rate swaps only after they became a standardized and liquid product. Central clearing for credit default swaps, which is now under consideration, also became relevant after a tremendous growth in market size (and the bad consequences during the recent financial crisis).

Since my paper illustrates a negative aspect of liquidity, it is related to Myers and Rajan (1998). In their model, greater asset liquidity reduces the firm's capacity to raise external finance because it reduces the firm's ability to commit to a specific course of action.

²For example, the London Clearing House clears over-the-counter interest rate swaps without being involved in the matching process and bargaining process. The Chicago Mercantile Exchange has also launched clearing services for some over-the-counter products. Following the recent and ongoing financial crisis, we may soon see central clearing for credit default swaps.

³Capital-based position limits, whose purpose is to make sure that members maintain positions within their financial capability, are different from speculative position limits. The latter are set by exchanges and regulators to prevent speculators from manipulating spot prices.

⁴Netting may also reduce collateral. However, some clearinghouses (e.g., the Hong Kong Futures Ex-

default may depend on activities in more than one market. Indeed, clearinghouses have recently moved toward more central clearing.⁵

Bernanke (1990) distinguishes between two roles of a clearinghouse: reducing the transactions cost of consummating agreed-upon trades (analogous to a bank that clears checks), and standardizing contracts by setting terms and format and by guaranteeing performance to both sides of trade (analogous to an insurance company).⁶ The mechanism in my paper has a more minimal role (it keeps track of reported trades), but the results remain even if we add other roles, such as guaranteeing performance. In addition, the model does not rule out multiple intermediaries.

The mechanism can also be interpreted as a regulator (e.g., a central bank). My theory suggests that to induce banks to report all their transactions voluntarily, the regulator may need to commit to keep these reports private. The theory also illustrates a connection between regulation and private-sector incentives to discipline. The regulator, who sets position limits, relies on firms in the private sector to discipline one another; that is, each firm makes sure that its trading partner reports the trade to the regulator. The theory implies that regulations that are too stringent may be counterproductive because they undermine private-sector incentives for agents to discipline one another.⁷

Contribution to existing literature. The paper contributes to the long line of literature on financial intermediation by illustrating a very minimal condition for an intermediary to be welfare improving and achieve second-best efficiency. The existing literature has focused on problems that arise because of asymmetric information regarding cash flows. In my paper the main problem is that an agent's history of transactions is private information. The existing literature has focused on the role of intermediaries in enhancing liquidity,

change Clearing Corporation) calculate margin on a gross basis rather than a net basis. Also while marking to market reduces the risk of default, when a counterparty's position is closed, there is a risk of not finding a new counterparty and remaining unhedged.

⁵For example, in 2004, the Chicago Mercantile Exchange (CME) fully integrated the clearing of all trades of the Chicago Board of Trade in addition to those of the CME. The CME has also developed cross-margin arrangements with other clearinghouses, so that margins can be calculated based on the total position.

⁶See also Telser and Higinbotham (1977) and Edwards (1983).

⁷While my model provides a novel rationale for regulatory secrecy, I do not present a full discussion of the costs and benefits of regulatory secrecy.

whereas I start with markets that are already liquid and show how an intermediary can help. Unlike Diamond (1984), I do not rely on diversification, and unlike in Townsend (1978), the intermediary arises when the fixed cost per trade is low rather than high.⁸

In a different framework, Bizer and DeMarzo (1992) and Parlour and Rajan (2001) study the effect of nonexclusivity on equilibrium interest rates and competition in credit markets.⁹ Bizer and DeMarzo assume that contracts entered in the past are observable and have a priority; additional contracts impose a negative externality on existing contracts because the agent's hidden effort affects his future income. In their setting a reporting mechanism such as the one in my paper cannot improve welfare. In contrast, I assume that previous contracts are not observable. Parlour and Rajan assume that intermediaries offer contracts simultaneously, and then a single borrower can accept any subset of these contracts. As in my paper, agents who strategically default do so on all the contracts they entered. In their model this can rule out entry even though competing lenders make positive profits. In my paper, this helps to sustain an equilibrium in which agents do not enter contracts secretly.

Paper outline. In Section 2, I present the economic environment. In Section 3 I solve for the optimal contract in the decentralized economy when contracts are exclusive (second best) and when contracts are assumed to be nonexclusive (third best). In Section 4, I add a central mechanism and show the main result: A position limit mechanism can achieve the second best, but a bulletin board mechanism may not. In Section 5, I show that the results remain even if we require renegotiation proofness (that is, a pair of agents cannot gain by coordinating their actions in a self-enforcing way) and in Section 6, I do comparative statics. Section 7 concludes, and the appendix contains omitted proofs.

⁸Brusco and Jackson (1999) show how a market maker can economize on the fixed costs of trading across periods. Madhavan (2000) summarizes the extensive literature on the effects of different trading mechanisms on liquidity provision. Gorton and Winton (2003) summarize the extensive literature on the role of banks. Carlin (2005) shows how an intermediary can enhance trade efficiency by becoming an uninformed broker between a buyer and a seller, who both have private information. Rubinstein and Wolinsky (1987) show how a middlemen can reduce search costs.

⁹See also Kahn and Mookherjee (1998), who study insurance contracts, Bisin and Rampini (2006), who study bankruptcy, and Bisin and Guaitoli (2004), who show that intermediaries can make positive profits by offering contracts that are not traded in equilibrium.

2 The Model

The model has a continuum of agents who enter mutual insurance contracts. Half of the agents are type 1 and half are type 2. Each contract is between a type-1 agent and a type-2 agent. Contracts are entered at date 0 and they specify date-1 payments contingent on the realized state. An agent can default strategically, but if he does so, he loses his future (date-2) income. In the benchmark case, each agent can enter at most one contract, which is enough to achieve the efficient allocation. In the main case, each agent can enter contracts with multiple counterparties without being observed doing so; i.e., an agent cannot observe contracts that his counterparty might have entered with other agents; in this case I say that an agent can enter nonexclusive contracts.

In more detail: There are three dates, $t = 0, 1, 2$, and one divisible good, called dollars, or simply cash. Uncertainty is modeled by assuming two states of nature, state 1 and state 2, one of which is realized at date 1. Agents are risk-neutral and obtain an expected utility of $E(c_0 + c_1 + c_2)$ from consuming c_0, c_1 and c_2 dollars at dates 0, 1, and 2, respectively. Agents are protected by limited liability, so $c_t \geq 0$ at each date.

At date 0 each agent has one dollar and an investment opportunity (project) that requires his human capital. The project yields nothing if transferred to another agent, so a bank cannot invest on behalf of agents as in Diamond and Dybvig (1983). Each project lasts for two periods. Consider first the project of a type-1 agent. The agent invests $I_1 \in [0, 1]$ at date 0. At date 1, the project yields εI_1 in state 1 ($\varepsilon > 0$) but requires an additional investment εI_1 in state 2. The additional investment must be made in full for the project to continue to date 2 and is called a negative cash flow. Finally, if the project continues to maturity (because it had a positive cash flow at date 1 or it had a negative cash flow and the additional investment was made in full), the project yields RI_1 dollars at date 2. Similarly, the project of a type-2 agent yields εI_2 in state 2 but requires εI_2 in state 1. If the project continues to maturity, it yields RI_2 dollars at date 2. (See Figure 1.) Note that there is no aggregate uncertainty at date 1: Half of the projects have positive cash flows

and half have negative cash flows.

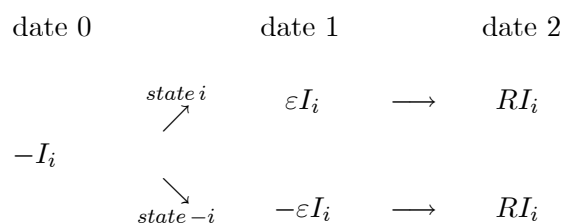


Figure 1: Project's cash flows for an agent of type i ($i=1,2$) if project operates to maturity.

The risk-free rate is normalized to be zero percent (i.e., there is a storage technology that gives one dollar at date $t + 1$ for every dollar invested at date t), and it is assumed that $R > 1 > \varepsilon$. The assumption $R > \varepsilon$ means that it is efficient to make the additional investment at date 1 if cash is available, and $R > 1$ means that in a world without frictions each project has a positive NPV; the NPV of i 's project is $(R - 1)I_i$. Finally, as explained in the next section, $\varepsilon < 1$ is a sufficient condition to ensure that entering bilateral contracts is preferred to autarky despite the moral hazard problems below.

Contracts can be contingent on the state that is realized at date 1. However, agents cannot commit to make payments. In particular,

Assumption 1 *An agent cannot commit to pay out of the project's final cash flows (RI_i).*

Assumption 2 *An agent cannot commit to pay out of the project's interim cash flows (εI_i).*

Assumption 1 implies that an agent with a negative cash flow cannot borrow at date 1 against the date-2 cash flows from his project. This is why agents enter forward contracts at date 0, according to which an agent with a positive cash flow transfers cash to an agent with a negative cash flow. Assumption 2 says that agents can default on such contracts. Both assumptions can be motivated by assuming that cash flows are not verifiable in court. The first assumption can also be motivated by assuming moral hazard as in Holmström and Tirole (1998), or by assuming that final cash flows are unobservable.

Enforcement technology. If an agent defaults (that is, does not pay what he promised in full), his project can be terminated. In this case the agent keeps current cash flows, but loses future cash flows. Liquidation values are zero at every date, consistent with the assumption that projects require human capital. In this model, it is optimal to terminate the project of a defaulting agent with probability one, even if it is possible to choose a probability less than one, and it is assumed that it is possible to commit to this closure policy. Allowing for additional penalties for default, such as spending time in prison, losing one’s reputation, or losing other sources of future income, does not alter the nature of the results (see Section 5). What’s important is that penalties impose a finite cost, rather than an infinite cost.

Collateral. Agents cannot post the projects’ assets as collateral, but they can post cash as collateral. Specifically, a pair of agents can open an escrow account, where they store cash through a third party who can commit not to divert it. Money placed in escrow is observable to both agents and can be contracted upon, but it is unobservable to agents who are not part of the bilateral contract (i.e., the agents can open a secret account). Posting collateral has an opportunity cost because agents are left with less cash to invest.

The last assumption in the benchmark model is that agents can divert the investment funds at date 0. Specifically,

Assumption 3 *The amount that an agent invests in his project (I_i) and the amount that an agent consumes are private information.*

Assumption 3 raises the risk of strategic default via “asset substitution”: An agent can consume his initial endowment instead of investing it and subsequently default at date 1, since he has no cash flows to pay from. Think of it as an agent diverting cash from one project (the original project) to another project (“consumption”) that yields some unobservable cash flow at date 0 and nothing afterward. As explained later, Assumption 3 is the reason agents may need to post collateral even in the benchmark case when contracts are exclusive.¹⁰

¹⁰ Assumption 3 is needed because without it, one can detect how many contracts an agent has entered by

Implicit here is that it is observable whether a project operates (it can be terminated upon default), but the level of investment (I_i) is private information. In addition, a project can operate even if $I_i = 0$. For example, an agent can go to work and keep his business open but effectively do nothing.

Nonexclusivity. To model the idea that agents can enter nonexclusive contracts (easily), I assume the following trading game: There are N trading rounds, all happen at date 0 before agents post collateral, invest in their projects, and/or consume (see Figure 2). In each trading round a fraction $\frac{1}{N}$ of agents (chosen randomly) arrive to trade for the first time, with an equal mass of both types; an agent does not know in what round he arrived. The sequence of events in each trading round is as follows: Agents who are present in the trading round are pairwise matched according to their types, and each pair enters a contract. Then each agent decides whether to enter additional contracts or stick with the contracts he has entered so far. An agent who wants to enter additional contracts stays for the next trading round to be matched with another counterparty. If there are no more trading rounds, the agent leaves the trading game and moves to the next stage, where he posts collateral. An agent who does not want to enter additional contracts also leaves the trading game. The sequence of events for an individual agent is in Figure 3.

Figure 2: Enter Figure 2 here

Figure 3: Enter Figure 3 here

Note that I do not try to model search and matching frictions. Instead, the game captures the idea that a counterparty may have entered contracts in the past and may enter additional contracts in the future, with none of these contracts being observable, as stated below. The trading environment also captures the idea of a large market, in which a deviation by one agent does not affect prices.

looking at I_i ; if an agent enters n_i contracts and each contract requires a collateral k_i , then $I_i = 1 - n_i k_i$, and we can infer n_i from I_i and k_i . If posting collateral did not come instead of investing, the assumption would not be necessary in this model.

Assumption 4 *An agent cannot observe contracts that other pairs of agents enter (either in the past or in the future).*

Assumption 4 has a few interpretations: Agents can enter contracts secretly,¹¹ trading is too fast for agents to keep track of a counterparty's history of transactions, or existing contracts are observable but not understood; an example is the complex derivative positions and off-balance-sheet transactions made by many hedge funds. As noted earlier, an agent cannot observe the amount of collateral that his counterparty posts with other agents; an agent who enters nonexclusive contracts opens a different escrow account with each counterparty.

In the equilibria considered in this paper, the outcome is that each agent enters a contract with the first agent he is matched with and leaves the trading game immediately afterward. Thus, along the equilibrium path the mass of type-1 agents equals the mass of type-2 agents in every trading round, and it is natural to assume equal bargaining power. In addition, since there is a continuum of agents in each round, a deviation by one agent or a finite number of agents does not alter the ratio of type 1 agents to type 2 agents in a given round; such a deviation cannot be detected by other agents, and it does not affect the probability of being matched.

For ease of exposition, I focus on the case in which N equals infinity, so an agent assigns a probability of zero to the event that he or his counterparty will not be able to enter additional contracts should either of them decide to do so. I also assume, for simplicity, that entering a contract involves no fixed cost. The case in which N is finite and/or entering a contract involves some fixed cost is postponed to Section 5.¹²

¹¹For example, according to the Wall Street Journal (August 25, 2005), "(hedge) funds sometimes move out of trades—'assign' them—without telling the bank that sold them the credit-derivative contract that their counterparty has changed." Another example is the Nigerian barge deal between Enron and Merrill Lynch, in which Enron allegedly arranged for Merrill Lynch to serve as a temporary buyer (of the barges) so as to make Enron appear more profitable than it was. According to a release by the Department of Justice (October 15, 2003), "Enron promised in a secret oral 'handshake' side-deal that Merrill Lynch would receive a return on its investment plus an agreed-upon profit..."

¹²In equilibrium, collateral requirements and/or position limits put an upper bound on the number of contracts that an agent can enter. Thus, the mass of agents present in each round is finite, even in the off-equilibrium-path event where a continuum of agents decide to stay for additional rounds.

3 Optimal contracts

I characterize optimal contracts as a solution to a planning problem. Without loss of generality, transfers can be made only at dates 0 and 1, all transfers at date 0 are in the form of collateral, and agents do not store on their own. Denote by x_i the amount of cash that an agent of type i transfers to the other agent in state i (an agent with a positive cash flow transfers cash to an agent with a negative cash flow), and by k_i the amount that he posts as collateral ($k_i \leq x_i$). I refer to $(I_i, k_i, x_i)_{i=1,2}$ as the *outcome*, or as the *contract*, and denote it by ψ . An agent can default only on the amount $x_i - k_i$.

The sequence of events is as follows: First, a central planner sets a contract. Then, agents play the trading game from Section 2, taking the contract as given. The sequence of events for an individual agent is in Figure 3. The agent decides whether to participate, how many contracts to enter, how much to invest, and how much to deliver. In Section 5, I show that the results remain even if a pair of agents can enter a contract different from the one suggested by the planner.

I say that a contract ψ can be *implemented* if there is a Perfect Bayesian Equilibrium (PBE) in which agents invest and deliver according to ψ .¹³ I restrict attention to symmetric equilibria, in which agents of the same type follow the same (pure) strategy. In this section I focus on two cases: First, I solve for the most efficient contract that can be implemented if each agent can enter at most one contract (second best). Then I focus on the case in which agents can enter nonexclusive contracts (third best).

A contract induces the following consumption stream for an agent of type i (assuming everyone follows the contract): At date 0, the agent consumes $1 - I_i - k_i$, which is his initial endowment minus what he invests and posts as collateral. At date 1, the agent consumes $k_i + \varepsilon I_i - x_i$ in state i , and $k_i + x_{-i} - \varepsilon I_i$ in the other state, denoted by $-i$. (In state i the agent pays what he promised using his collateral and his project's cash flows; in state $-i$ the agent makes the additional investment in his project using his collateral and the

¹³Without loss of generality, I rule out the case in which the planner suggests a contract with the intention that each agent will enter it more than once.

payment received from his counterparty.) At date 2, the agent consumes RI_i . The agent's expected utility is

$$\begin{aligned} U_i(\psi) &\equiv 1 - I_i - k_i + \frac{1}{2}(k_i + \varepsilon I_i - x_i) + \frac{1}{2}(k_i + x_{-i} - \varepsilon I_i) + RI_i \\ &= 1 + (R - 1)I_i + \frac{1}{2}(x_{-i} - x_i). \end{aligned} \quad (1)$$

When a contract is symmetric, I will sometimes drop the index i . The contract is *feasible* if: (i) $I_i \geq 0$; (ii) $x_i \geq k_i \geq 0$; and (iii) the amount that agents consume at each date and state is nonnegative; that is,

$$1 - I_i - k_i \geq 0 \text{ for every } i \in \{1, 2\}, \quad (2)$$

$$k_i + \varepsilon I_i - x_i \geq 0 \text{ for every } i \in \{1, 2\}, \quad (3)$$

and

$$k_i + x_{-i} - \varepsilon I_i \geq 0 \text{ for every } i \in \{1, 2\}. \quad (4)$$

Note that equation (4) implies that each agent has enough cash to make the additional investment; otherwise, agents would not enter contracts to begin with.

Since the two types of agents are identical ex-ante and have equal proportion, it is natural to assume that the planner's objective is to maximize the unweighted sum of utilities $U_1(\psi) + U_2(\psi)$. This is equivalent to maximizing $I_1 + I_2$. The solution to this problem (*first-best*) is $I_1 = I_2 = 1$, $k_1 = k_2 = 0$, and $x_1 = x_2 = \varepsilon$. In the first best agents do not post collateral, and the utility for each agent is R .

3.1 Exclusive contracts (second best)

In the second best we need to make sure that: (i) agents have the incentives to invest and make the transfers suggested by the planner (incentive compatibility); and (ii) each agent prefers the proposed contract to autarky (participation).

Participation. The participation constraint is

$$U_i(\psi) \geq U_A \text{ for every } i \in \{1, 2\}, \quad (5)$$

where U_A denotes the utility an agent obtains in autarky. In autarky, an agent has two options. The first is to self-insure by investing I and storing $s = 1 - I$ so that $s = \varepsilon I$. In this case, the agent can continue his project in both states, $I = \frac{1}{1+\varepsilon}$, and the agent's utility is $s + RI = \frac{R+\varepsilon}{1+\varepsilon}$. The second option is to invest $I = 1$, store nothing, and obtain a utility of $\frac{R+\varepsilon}{2}$. In this case the agent cannot continue his project when he realizes a negative shock. The first alternative is preferred if and only if $\varepsilon < 1$, as assumed here. Thus,

$$U_A = \frac{R + \varepsilon}{1 + \varepsilon}. \quad (6)$$

The assumption $\varepsilon < 1$ is sufficient to guarantee that entering bilateral contracts is preferred to autarky because when $\varepsilon < 1$ a pair of agents can do strictly better than autarky by allocating all the cash stored to the agent with the negative cash flow, so that each agent can invest more and store less.¹⁴

Incentives to make payments. Suppose an agent invested I , posted a total amount of collateral k , and promised a total amount x in the state in which he obtains εI from his project. If $k + \varepsilon I < x$, the agent does not have enough cash to deliver the full amount. In this case, it is optimal for him to pay nothing because if he makes a partial payment he still loses his project. If $k + \varepsilon I \geq x$, the agent has the capacity to pay what he promised. In this case, it is optimal for him to deliver the full amount because otherwise he gains $x - k \leq \varepsilon I$, but loses $RI > \varepsilon I$. Denote by d whether an agent delivers ($d = 1$) or not ($d = 0$). The optimal delivery rule is

$$d(I, k, x) = \begin{cases} 1 & \text{if } k + \varepsilon I \geq x \\ 0 & \text{if otherwise.} \end{cases} \quad (7)$$

Incentives to invest. Denote by $U_i(I'_i | \psi)$ the utility for an agent of type i if he invests $I'_i \in [0, 1 - k_i]$ and follows the optimal delivery rule, given that he entered the contract $\psi = (I_i, k_i, x_i)_{i=1,2}$ and his counterparty follows the contract. Denote $d(I'_i) \equiv d(I'_i, k_i, x_i)$.

¹⁴In other words, the symmetric contract (I, x, k) that satisfies $\varepsilon I = 2k$, and $x = k = 1 - I$ is strictly preferred to autarky.

Then

$$\begin{aligned}
U_i(I'_i|\psi) &= 1 - k_i - I'_i \\
&+ \frac{1}{2}[\varepsilon I'_i - d(I'_i)(x_i - k_i) + d(I'_i)RI'_i] \\
&+ \frac{1}{2}[k_i + x_{-i} + \beta_i(I'_i)(R - \varepsilon)I'_i],
\end{aligned} \tag{8}$$

where $\beta_i(I'_i)$ denotes whether the agent has enough cash to make the additional investment; that is,

$$\beta_i(I'_i) = \begin{cases} 1 & \text{if } k_i + x_{-i} \geq \varepsilon I'_i \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

The first line in equation (8) shows the amount the agent consumes at date 0. The second line shows the total amount the agent consumes at dates 1 and 2 if he realizes a positive cash flow, and the third line shows the amount consumed when the agent realizes a negative cash flow. Note that $U_i(I_i|\psi) = U_i(\psi)$.

The incentive constraint is that for $i \in \{1, 2\}$,

$$U_i(I_i|\psi) \geq U_i(I'_i|\psi) \text{ for every } I'_i \in [0, 1 - k_i]. \tag{10}$$

The second-best problem is to find a feasible contract that maximizes $I_1 + I_2$ subject to equations (5) and (10).

Equation (10) can be replaced with

$$U_i(I_i|\psi) \geq U_i(0|\psi). \tag{11}$$

In other words, it is enough to focus on deviations in which an agent invests nothing in his project and then defaults when he needs to make a payment. Intuitively, an agent who plans to default is better off consuming his initial endowment rather than investing it and losing it upon default.¹⁵

The incentive constraint reduces to

$$\frac{1}{2}(x_i - k_i) \leq (R - 1)I_i \text{ for every } i \in \{1, 2\}. \tag{12}$$

¹⁵A formal proof is in Lemma 3 in the appendix.

Intuitively, the expected gain from not delivering the promised amount (left-hand side) must be less than or equal to the expected loss from not investing in one's project (right-hand side).

The second-best problem reduces to finding a feasible contract that maximizes $I_1 + I_2$ subject to equations (5) and (12). This is a linear programming problem. When $R \geq 1 + \frac{1}{2}\varepsilon$, the first-best contract satisfies all the constraints and is a unique solution. In this case, the incentive constraint is not binding. When $R < 1 + \frac{1}{2}\varepsilon$, equation (12) implies equation (3), and the solution is obtained by solving equations (2), (4), and (12) with equalities. In this case the contract is such that agents do not consume at date 0, each agent has just what he needs to make the additional investment but not more, and each agent is indifferent between following the contract and planning a strategic default. Collateral is needed in this case to prevent a strategic default, in which an agent consumes his initial endowment instead of investing it.

Proposition 1 (second best) *If $R \geq 1 + \frac{1}{2}\varepsilon$, the second-best contract equals the first-best contract. Otherwise, the second-best contract is given (uniquely) by $k_1 = k_2 = k$, $x_1 = x_2 = \varepsilon - (1 + \varepsilon)k$, and $I_1 = I_2 = 1 - k$, where $k = \frac{\varepsilon - 2(R-1)}{\varepsilon - 2(R-1) + 2}$.*

Denote the second-best contract by $\psi_{sb} \equiv (I_{sb}, k_{sb}, x_{sb})$. Equation (1) and Proposition 1 imply that

$$U_i(\psi_{sb}) = R - (R - 1)k_{sb}. \quad (13)$$

The first term is the agent's first-best utility, and the second term is the opportunity cost of collateral: By posting collateral, agents forgo investing in their positive NPV projects.

3.2 Nonexclusive contracts (third best)

In the third best agents are not restricted to enter only one contract. A possible deviation for an agent of type i (assuming every other agent enters one contract and follows it) is to enter as many contracts as he can, invest nothing in his project, and eventually default on all contracts. This is the most profitable deviation. Since each contract requires collateral,

and the agent has only one dollar, the number of contracts that the agent can enter (n_i) must satisfy $n_i k_i \leq 1$. The expected utility for the deviating agent is

$$\begin{aligned}\bar{U}_i(\psi, n_i) &\equiv (1 - n_i k_i) + \frac{1}{2} n_i (k_i + x_{-i}) + \frac{1}{2} (0) \\ &= 1 + \frac{1}{2} n_i (x_{-i} - k_i).\end{aligned}\tag{14}$$

The expression in the first brackets in the first line is what the agent consumes at date 0. The other two expressions in the first line represent the expected amount consumed at date 1: In one state the agent receives back all the collateral he posted plus a payment from each counterparty. In the other state, in which the agent needs to deliver, the agent loses his collateral and ends up with nothing due to his limited liability.

The incentive constraint is therefore

$$U_i(\psi) \geq \bar{U}_i(\psi, n_i)\tag{15}$$

for every $i \in \{1, 2\}$ and every integer n_i such that $n_i \leq \frac{1}{k_i}$. Equation (15) reduces to

$$\frac{1}{2}(x_i - k_i) + \frac{1}{2}(n_i - 1)(x_{-i} - k_i) \leq (R - 1)I_i\tag{16}$$

(using equations (1) and (14), and rearranging terms). When $n_i = 1$, equation (16) is the same as in the case of exclusive contracts (equation (12)). The extra term when $n_i > 1$ is the expected net payoff from entering additional contracts and not delivering on them: In one state the agent collects a payment from his counterparty, whereas in the other state the agent loses his collateral.

Denote by Ψ the set of contracts that satisfy the incentive constraint (equation (16)) and the participation constraint. A feasible contract ψ can be implemented if and only if $\psi \in \Psi$. To see why the incentive constraint is necessary, consider an equilibrium in which every agent enters ψ and follows it. An agent cannot observe what other agents have done in the past, but he must believe that all the agents who are present in the current round have just showed up for trade. These are the only beliefs that are consistent with the equilibrium path because agents who showed up in previous rounds must have entered one

contract and left the trading process.¹⁶ Given these beliefs and assuming that all other agents follow their equilibrium strategies from now on, an agent who shows up for trade for the first time expects to obtain either $U_i(\psi)$ or $\bar{U}_i(\psi, n_i)$, depending on the action he chooses. The incentive constraint is necessary to ensure that he will follow the contract.¹⁷

The third-best problem is to choose a feasible contract $\psi \in \Psi$ that maximizes $I_1 + I_2$. To ensure that a solution exists, I drop the restriction that n_i be an integer. A micro foundation for this can be obtained by assuming that instead of one economy, there are an infinite number of economies corresponding to the interval $(0, 1]$. Agents in economy $\mu \in (0, 1]$ have an initial endowment of μ , and they enter the contract $\mu\psi \equiv (\mu I_i, \mu k_i, \mu x_i)_{i=1,2}$. The economy to which an agent belongs to and an agent's endowment are private information. When an agent first shows up to trade, he must trade in his original economy, but afterward an agent with an endowment e can switch back and forth among any of the economies in the interval $(0, e]$; that is, an agent can say that he has less than what he has, but he cannot say that he has more.¹⁸

Denote the (unique) solution to the third-best problem by ψ_{tb} . This contract is symmetric. I and x depend on k as in Proposition 1, and k is obtained from equation (16) with $n_i = \frac{1}{k_i}$. The third-best contract requires more collateral than the second-best contract and provides agents with a lower utility.

Proposition 2 (third best) *There is a unique contract that solves the third-best problem.*

This contract is given by $k_1 = k_2 = k$, $x_1 = x_2 = \varepsilon - (1 + \varepsilon)k$, and $I_1 = I_2 = 1 - k$, where

¹⁶Note that an agent does not need to form beliefs about the whole history of the game. It is enough to form beliefs about the number of contracts that agents present in the same round have entered.

¹⁷To show that the incentive constraint is sufficient, we can come up with the following strategies: Participate if $U_i(\psi) \geq U_A$. If ψ satisfies equation (16) enter one contract and leave the trading process; otherwise, continue to enter contracts until you run out of collateral. Finally, if you entered n_i contracts, choose $I'_i \in [0, 1 - n_i k_i]$ to maximize $U_i(I'_i | n_i \psi)$, where $n_i \psi$ denotes the contract $(I_i, n_i k_i, n_i x_i)_{i=1,2}$; deliver according to the optimal deliver rule. In the off-equilibrium-path event in which equation (16) holds, but an agent has entered $n > 1$ contracts on which he can deliver, the agent will base his decision by comparing his utility from entering n contracts and delivering, to entering as many contracts as he can and defaulting on all contracts.

¹⁸The parameter δ introduced in Section 5 is scaled to $\mu\delta$ in economy μ . The parameter M is scaled to μM if an agent's endowment is μ . If n_i is restricted to be an integer, a solution may not exist because the set of feasible contracts that satisfy equation (16) may be open (because n_i is not a continuous function of k_i).

k solves (16) with equality; that is, $k = \frac{1}{4r} \left(b - \sqrt{b^2 - 8r\varepsilon} \right)$, $r = R - 1$ and $b = 2 + \varepsilon + 2r$.

Implementation. A central planner is not necessary to implement the third-best contract, or any other feasible contract in Ψ . For example, we can assume that when two agents are matched, they offer contracts simultaneously and a contract is entered only if both agents offer the same contract. Alternatively, we can assume that one agent offers a contract and the other agent accepts or rejects. In both cases, there is a PBE in which agents enter ψ .¹⁹

The dual role of collateral. For a symmetric contract (I, k, x) , like the third best, equation (16) reduces to

$$x \leq k + \frac{2(R-1)I}{n}, \quad (17)$$

where $n = \frac{1}{k}$. When an agent posts k dollars as collateral, the amount of cash that he can credibly promise (x) increases by more than k . First, the agent cannot default on the amount of cash that he posted as collateral (first term in equation (17)). Second, the fact that the contract requires collateral limits the number of contracts (n) that the agent can enter. This makes the threat of losing future cash flows valuable in backing promises (second term).

4 The mechanism design problem

4.1 The message game

In this section—which contains the main results—I extend the trading game by allowing agents to exchange messages with some central mechanism, which can observe nothing but the messages sent to it. I refer to the extended game as the *message game* and show that a well-designed message game can implement the second best. Since the second-best contract is the best contract that can be implemented given assumption 1-3, a mechanism that implements the second best is optimal.

¹⁹In the first case, it is suboptimal to offer $\psi' \neq \psi$ because a contract is entered only if both agents offer the same contract. In the second case we can sustain an equilibrium in which all agents enter ψ , by assuming that upon seeing an offer $\psi' \neq \psi$, an agent assumes that his counterparty will default because the counterparty has already promised more than he has. Since this is an out-of-equilibrium event, any beliefs may be assigned.

Denote the set of agents by Υ . A message from agent $i \in \Upsilon$ to the mechanism is a pair (i, j) , where i denotes the identity of the agent sending the message and $j \in \Upsilon$ denotes the identity of his counterparty. Later, when I allow agents to enter contracts different from the one suggested by the planner, a message also includes the contract. When agent i sends the message above, I say that agent i reports agent j and/or that agent i reports the contract. An agent is not obliged to send a message to the mechanism, but if he does, he cannot lie; for example, he needs to send a copy of the signed contract.

To make the analysis more interesting, I assume that an agent who reports a contract incurs a small reporting fee θ . Thus, an agent reports only if he strictly prefers to do so. To keep the analysis simple, I focus on the case in which θ approaches zero and exclude it from the expressions below.

Denote by M_τ the set of all messages received by the mechanism up to trading round τ . The mechanism does two things: First, the mechanism notifies an agent if another agent reported him; formally, if agent i sends the message (i, j) to the mechanism, the mechanism sends the message (i, j) to agent j . Second, at the beginning of each trading round, the mechanism makes a public announcement that is observable at no cost to every agent who is present in the round. The announcement in the beginning of round τ uses the information gathered in the first $\tau - 1$ rounds and is denoted by $P(M_{\tau-1})$.²⁰

As in the previous section, a planner sets a contract (ψ) , and agents play a game taking the contract as given. The message game (denoted by G_0) is an extension of the trading game in Section 2. As before, there are N trading rounds. The sequence of events in round τ is as follows:

1. The mechanism makes a public announcement $P(M_{\tau-1})$.
2. Agents are pairwise matched as in Section 2. If both agents want to enter a contract, they enter ψ . Otherwise, the agents do not enter a contract and each agent moves to

²⁰To make a public announcement, the mechanism does not need to know the identities of agents who are present in a given round. Also note that the nature of the results does not depend on whether the decision to report a contract is unilateral (as assumed here) or bilateral (as assumed in previous drafts of this paper).

the next trading round to be matched with a different agent.

3. Each agent can send a message to the mechanism (all messages are sent simultaneously).
4. The mechanism notifies an agent if another agent reported him.
5. Each agent decides whether to stay for the next trading round (to enter an additional contract) or leave the trading process. If he leaves, he moves to the next stage, where he posts collateral, invests, and consumes.

I focus on two types of mechanisms: (1) A *bulletin board mechanism*, where the mechanism makes public all the information it has, i.e., $P(M_\tau) = M_\tau$. (2) A *position limit mechanism*, where the mechanism announces the identities of agents who, according to their counterparties, have entered the contract ψ at least L times. Formally, $P(M_\tau) = \{i \in \Upsilon : n_\tau(i) \geq L\}$, where $n_\tau(i)$ is the number of agents who reported agent i up to trading round τ ; that is, $n_\tau(i)$ is the number of elements in the set $\{(i_1, i_2) \in M_\tau : i_2 = i\}$. If agents can enter any contract, as assumed later, the mechanism updates $P(M_\tau)$ only if the message includes the contract set by the planner. When $i \in P(M_\tau)$, I say that agent i is *on the list*.

The mechanism—like any other agent—cannot observe contracts that agents enter without reporting. The mechanism keeps track only of reported contracts. The main result is that a position limit mechanism can implement the second best, but only if the position limit is set appropriately; in some cases, position limits must be nonbinding in equilibrium. In contrast, a bulletin board mechanism cannot always implement the second best.

4.2 An optimal mechanism

Start with a position limit mechanism. A position limit mechanism is optimal if there exists a position limit $L \geq 1$ for which there is a PBE in which every agent enters the second-best contract exactly once and reports it to the mechanism. In such a PBE, strategies must

be such that an agent who sees that his counterparty's name is on the list does not enter a contract with him and instead waits for the next round. If this is not the case, the list is meaningless and we are back to the situation in Section 3, in which the second best could not be implemented. Note that by waiting for the next round, an agent loses nothing because he can enter the same contract with another agent.²¹

Thus, given that everybody reports, an agent who plans a strategic default can enter at most L contracts. To implement the second best, we must have

$$U_i(\psi_{sb}) \geq \bar{U}_i(\psi_{sb}, L) \text{ for every } i \in \{1, 2\}. \quad (18)$$

Equation (18) is obtained from equation (15) (the incentive constraint) when we substitute the second-best contract and $n_i = L$. When the second best equals the first best ($R \geq 1 + \frac{1}{2}\varepsilon$), equation (18) reduces to $L \leq L_2 \equiv \frac{2(R-1)}{\varepsilon}$ (using Proposition 1). Otherwise, the equation reduces to $L \leq 1$, so we must have $L = 1$.

The more interesting part is to induce agents to report their trades; remember, an agent reports only if he strictly prefers to do so. Below, I show that reporting can be induced by the threat of default. However, this threat is credible only if the position limit is high enough.

To see how it works, consider the following continuation game (denoted by G_1) played by two agents who entered a contract. The game has two stages: First, the two agents (referred to as agent 1 and agent 2) choose simultaneously whether to report the contract. Second, after observing the outcome of the first stage, each agent chooses (simultaneously) whether to plan a strategic default. Planning a strategic default (or in short, "default") means that an agent enters as many contracts as he can, planning to default on all of them. Denote by ρ_i whether agent $i \in \{1, 2\}$ reports the contract ($\rho_i = 1$) or not ($\rho_i = 0$), and by $d_i \in \{0, 1\}$ whether the agent defaults ($d_i = 0$) or not ($d_i = 1$). Note that d_i depends on (ρ_1, ρ_2) because an agent knows whether the other agent reported him or not.

²¹When agents can have different endowments, the position limit for an agent from an economy μ is μL . A message to the mechanism includes not only the agents' identities but also the economy in which the contract was entered. An agent's position limit depends on the economy from which he reported his first contract.

Payoffs in G_1 depend on the number of contracts that an agent who plans to default can enter. This number depends on the position limit and on whether the agent's counterparty reported him, and is given by

$$n(L, \rho_{-i}) = \begin{cases} L & \text{if } \rho_{-i} = 1 \\ L + 1 & \text{if } \rho_{-i} = 0. \end{cases} \quad (19)$$

If a counterparty reports the current contract, and since all other agents report, the agent can enter $L - 1$ additional contracts before his name is on the list. The agent therefore ends up with a total of L contracts. If a counterparty does not report, the agent can enter L additional contracts for a total of $L + 1$.²²

Payoffs for the game are in Figure 4; the payoff for agent i is denoted by $U_i(\rho_1, \rho_2, d_1, d_2)$. If both agents deliver (first row), each agent obtains $U_i(\psi_{sb})$. If one agent delivers and the other agent does not (rows 2 and 3), the agent who delivers obtains the autarkic utility less the expected net payment to his counterparty ($U_A - \frac{1}{2}(x_{sb} - k_{sb})$), and the other agent obtains $\bar{U}_i(\psi_{sb}, n(L, \rho_{-i}))$. Finally, if both agents default (row 4), the contract they entered does not count, and the utility for each agent is $\bar{U}_i(\psi_{sb}, n(L, \rho_{-i}) - 1)$, which equals $\bar{U}_i(\psi_{sb}, n(L, \rho_{-i})) - \frac{1}{2}(x_{sb} - k_{sb})$. (To simplify notation, I dropped the index i in Figure 4.)

$U_1(\rho_1, \rho_2, d_1, d_2)$	$U_2(\rho_1, \rho_2, d_1, d_2)$	d_1	d_2
$U(\psi_{sb})$	$U(\psi_{sb})$	1	1
$U_A - \frac{1}{2}(x_{sb} - k_{sb})$	$\bar{U}(\psi_{sb}, n(L, \rho_1))$	1	0
$\bar{U}(\psi_{sb}, n(L, \rho_2))$	$U_A - \frac{1}{2}(x_{sb} - k_{sb})$	0	1
$\bar{U}(\psi_{sb}, n(L, \rho_2)) - \frac{1}{2}(x_{sb} - k_{sb})$	$\bar{U}(\psi_{sb}, n(L, \rho_1)) - \frac{1}{2}(x_{sb} - k_{sb})$	0	0

Figure 4: Payoffs in the continuation game after a pair of agents enter a contract.

I solve the game backwards. If the outcome of the first stage is that both agents report ($\rho_1 = \rho_2 = 1$), choosing $d_1 = d_2 = 1$ is an equilibrium if and only if $U_i(\psi_{sb}) \geq \bar{U}_i(\psi_{sb}, n(L, \rho_{-i}))$, for $i \in \{1, 2\}$. This is equivalent to equation (18) above. Consider now the first stage. Reporting can be sustained by the threat that a counterparty will default if he sees that the agent did not report the contract. The next lemma shows that this threat is credible only if the position limit is high enough.

²²The number of contracts that an agent can enter does not depend on collateral requirements, since in an equilibrium that implements the second best, the collateral constraint is not binding.

Lemma 1 (1) If $R < 1 + \frac{1}{2}\varepsilon$, choosing $d_i = 0$ after observing $\rho_i = 1$ and $\rho_{-i} = 0$ is a credible threat if and only if $L \geq 1$. (2) If $R \geq 1 + \frac{1}{2}\varepsilon$, the threat above is a credible if and only if $L \geq L_1 \equiv \min(L_2 - 1, \frac{2(u_A - 1)}{\varepsilon})$.

Theorem 1 then follows.

Theorem 1 (1) If $R < 1 + \frac{1}{2}\varepsilon$, a position limit mechanism can implement the second best if and only if $L = 1$. (2) If $R \geq 1 + \frac{1}{2}\varepsilon$, a position limit mechanism can implement the second best if and only if $L \geq 1$ and $L \in [L_1, L_2]$.

Intuitively, the position limit cannot be too high and it cannot be too low. Too high a position limit induces agents to enter too many contracts planning to default on all of them. Too low a position limit induces agents to deviate by not reporting. By not reporting, an agent increases the counterparty's gain from strategic default by allowing him to enter more contracts than the position limit. However, if the position limit is too low, the counterparty will never default, whether the agent reports him or not. In this case, an agent will not report to save the reporting fee.²³

When $R > 1 + \varepsilon$, the lower bound in Theorem 1 is strictly greater than one. Thus, the position limit must be greater than one, even though in equilibrium every agent enters only one contract.

Corollary 1 If $R > 1 + \varepsilon$, the position limit must be nonbinding in equilibrium.

Example 1 Suppose $R = 1.42$ and $\varepsilon = 0.2$, so the second best equals the first best (Proposition 1); i.e., each agent invests \$1 and promises to pay \$0.2. In this case $L_2 = 4.2$, and a position limit mechanism can implement the second best if and only if $L \in [3.2, 4.2]$. Thus, the position limit must be nonbinding in equilibrium.

For example, if $L = 4$, an agent has an incentive to report because if he does not report his counterparty can enter 4 additional contracts, for a total of 5. But then the counterparty will plan a strategic default because by entering 5 contracts, and defaulting on all,

²³Note that there is always an integer that belongs to the interval $[L_1, L_2]$.

he obtains $1 + \frac{1}{2} \times 5 \times 0.2 = 1.5$, which is more than the 1.42 he obtains by entering only one contract and delievering on it. If, however, the position limit is only 3 and the agent does not report, the counterparty can enter a total of only 4 contracts, thereby obtaining $1 + \frac{1}{2} \times 4 \times 0.2 = 1.4$. Since this is less than 1.42, the counterparty will not default even if the agent does not report the contract. But then the agent is better off not reporting and saving the reporting fee.

Theorem 1 also implies that for some parameter values, a bulletin board mechanism cannot implement the second best. To see why, suppose, for example, that $R > 1 + 1.5\varepsilon$, so the second best equals the first best and $L_1 \geq 2$. Suppose, in addition, that $\varepsilon > \frac{1}{2}$, so an agent who entered two contracts (promising $2\varepsilon > 1$) has no choice but to default at date 1. (This follows because for any $I \in [0, 1]$ and $k = 1 - I$, the amount of cash that the agent has at date 1 satisfies $\varepsilon I + k \leq 1$.) Then once it becomes public that an agent has entered a contract, no other agent will enter a contract with him. However, we know from the analysis above that to induce reporting when $L_1 \geq 2$, an agent must be allowed to report at least two contracts.

Corollary 2 *For some parameter values, a bulletin board mechanism cannot implement the second best.*

Remarks:

1. It is suboptimal to report a contract from a previous round.
2. It is crucial that an agent learns whether a counterparty reported him. Otherwise, a counterparty can deviate by not reporting, with the agent still believing that the counterparty reported the contract.
3. The nature of the results remains even if a contract can include a clause according to which the contract is void if an agent signing the contract gets on the list. When $R < 1 + \frac{1}{2}\varepsilon$ (so according to Theorem 1, $L = 1$) agents will not include this clause.

Otherwise, agents may include the clause. This means that if the position limit is L , an agent who plans to default can enter any number of contracts as long as it is strictly less than L . But we know from Lemma 1 that to induce reporting an agent must be able to enter at least L_1 contracts. This means we must have $L_1 < L$, and the interval in Theorem 1 becomes $(L_1, L_2]$.

4. We implicitly assumed in this Section that it is impossible to void a contract if one agent does not report it; in other words, we assumed that the messages from the mechanism are not verifiable in court. If we relax this assumption, the lower bound in Theorem 1 changes to $L_1 = 1$ (no matter what the parameters of the problems are) and position limits need not be nonbinding. The result on nonbinding position limits (and the implication in Corollary 2) still holds, however, when we require renegotiation proofness in the next section.

4.3 Implementation

The optimal mechanism can be implemented by an intermediary with a very minimal role. The intermediary needs to set position limits, let agents register (report) their contracts and make public the list of agents who have reached the position limit. The closest real-world example is a clearinghouse. If the second best equals the first best, no collateral is required. Otherwise, the intermediary must require collateral, but less than the amount necessary to sustain trade without an intermediary.

What if the intermediary guarantees payments? The paper focuses on a minimal role, which is sufficient to implement the second best, but the main results remain even if the intermediary has some additional roles. For example, the results remain if in addition to setting position limits and keeping track of reported trades, the intermediary also becomes a central counterparty that guarantees payments; this is one of the roles of a clearinghouse. In my model, the intermediary should guarantee payments only if the contract is reported to it (so the intermediary can monitor), and only if an agent has not reached the position limit. Theorem 1 still holds: Position limits cannot be too stringent. Also note that since

default never happens in equilibrium, the intermediary does not need to have any capital to make his guarantee credible. In the off-equilibrium event in which an agent enters more than one contract and defaults, the intermediary will default as well. The intermediary can prevent this type of default by setting aside some capital, but this is not necessary in this model.

Multiple intermediaries. One intermediary can implement the second best in this paper, but the results do not rule out multiple intermediaries. For example, a mechanism that sets a position limit $L = 4$ can be implemented by one intermediary that sets a position limit of 4, or by four intermediaries, each setting a position limit of one.²⁴ To see how it works, adjust the trading game by assuming up to L locations. Each location has its own intermediary, and each intermediary can observe only the contracts reported to it. Each agent shows up for trade in a randomly chosen location. Initially, an agent must trade in the location where he showed up, but if an agent decides to stay for more rounds (to enter more contracts and default on all), he can switch back and forth among the different locations. Pairwise matching in each location is as in the original game, and if an agent chooses to report, he must report to the intermediary in the location where he is. This extended game has a PBE in which each agent enters one contract and reports it to the intermediary in his original location. For this to work, an agent must observe whether his counterparty has reached the position limit in the location where the agent currently is, but not in other locations. Otherwise, an agent can infer how many contracts his counterparty has entered (which resembles a bulletin board mechanism), and not only whether the counterparty has reached the limit. Such a restriction follows, for example, if an agent in a given location needs to incur some cost to see whether his counterparty is on the list in another location. In equilibrium, no agent will incur this cost because given the equilibrium beliefs (a counterparty has just showed up for trade), an agent gains nothing by checking the lists in other locations.

²⁴More generally, L can be implemented by N_j intermediaries, such that intermediary j sets a position limit $l_j \geq 1$, and $\sum_{j=1}^n l_j = L$.

5 Extensions

5.1 Renegotiation proofness

So far we assumed that a pair of agents cannot enter a contract different from the one set by the planner (ψ). Now I relax this assumption using the concept of renegotiation proofness. Loosely speaking, ψ is renegotiation proof if it is impossible to find a contract $\psi' \neq \psi$ such that a pair of agents would like to enter ψ' instead of ψ , given that all other pairs of agents stick with ψ . Reporting ψ is renegotiation proof if a pair of agents cannot gain by entering ψ without reporting or by entering ψ' with or without reporting. A formal definition is in the appendix.

Requiring renegotiation proofness does not alter the nature of the results. Proposition 3 relates to the decentralized setting and Proposition 4 relates to the optimal position limit mechanism.

Proposition 3 (1) *The third-best contract is renegotiation proof in the decentralized trading environment.* (2) *The third-best contract is the only contract in Ψ that is both symmetric and renegotiation proof in the decentralized trading environment.*

Proposition 4 (1) *If $R < 1 + \frac{1}{2}\varepsilon$, a position limit mechanism can implement the second best in a renegotiation proof way if and only if $L = 1$.* (2) *If $R \geq 1 + \frac{1}{2}\varepsilon$, a position limit mechanism can implement the second best in a renegotiation proof way if and only if $L \geq 1$ and $L \in (L_2 - 1, L_2]$.*

To see why Proposition 4 is true, start with the case in which agents are restricted to enter the second-best contract, so the only choice is whether to report or not. Lemma 2 shows under what conditions it is possible to sustain a deviation in which a pair of agents enter ψ_{sb} without reporting. Such a deviation is possible if and only if choosing $d_1 = d_2 = 1$ after observing $\rho_1 = \rho_2 = 0$ is an equilibrium in the continuation game G_1 , defined in the previous section. For renegotiation proofness, we want to rule out the deviation above.

Lemma 2 Consider the continuation game G_1 and assume that agents are restricted to entering the second-best contract. (i) If $R < 1 + \frac{1}{2}\varepsilon$, choosing $d_1 = d_2 = 1$ after observing $\rho_1 = \rho_2 = 0$ cannot be sustained in equilibrium. (ii) If $R \geq 1 + \frac{1}{2}\varepsilon$, choosing $d_1 = d_2 = 1$ after observing $\rho_1 = \rho_2 = 0$ can be sustained in equilibrium if and only if $L \leq L_2 - 1$.

Consider now the case in which a pair of agents who want to deviate are not restricted to entering the second-best contract. Reporting $\psi \neq \psi_{sb}$ is suboptimal because the mechanism counts only the contract set by the central planner. Consider entering $\psi \neq \psi_{sb}$ without reporting. The agents save the reporting fee, but to prevent default the contract must require that agents post more collateral than in the second best. If the reporting fee is small enough, the extra cost of collateral outweighs the benefits of not reporting.²⁵

5.2 Trading cost and penalty for default

The nature of the results remain if we alter $U_i(\psi)$ (the agent's utility if he delivers) and $\bar{U}_i(\psi, n_i)$ (the agent's utility if he plans a strategic default) as follows:

$$U_i(\psi) = 1 + (R - 1)I_i + \frac{1}{2}(x_{-i} - x_i) - \delta \quad (20)$$

$$\bar{U}_i(\psi, n_i) = 1 + \frac{1}{2}n_i(x_{-i} - k_i) - \frac{1}{2}M - n_i\delta. \quad (21)$$

The parameter δ is a fixed cost per trade, and M is a fixed penalty upon default (which happens with probability $1/2$), both measured in utility terms. You can think of δ as the time and effort involved in entering a contract, and you can think of M as the cost of spending time in prison, loss of reputation, or loss of future income.

With the added parameters, the second best equals the first best if $R \geq 1 + \frac{1}{2}(\varepsilon - M)$. Otherwise, the level of collateral in Proposition 1 changes to $k = \frac{\varepsilon - 2(R-1) - M}{\varepsilon - 2(R-1) + 2}$; that is, with a higher penalty for default, less collateral is needed. For the other results to hold, we need to put some restrictions on the fixed cost per trade δ . First, δ must be small enough so that the participation constraint is satisfied. Second, δ must be small enough so that the third-best

²⁵Implicit here is that the reporting fee is zero. Otherwise, there exists $L'_1 > L_1$, such that the cost of collateral outweighs the fee if and only if $L \geq L'_1$, and the interval in Theorem 1 becomes $[L'_1, L_2]$. This is because with a higher position limit, strategic default becomes easier and more collateral is needed.

contract does not equal the second best; otherwise, the mechanism is redundant.²⁶ When these restrictions are satisfied, the third best is as in Proposition 2 with $b = 2 + \varepsilon' + 2r + M$, and $\varepsilon' \equiv \varepsilon - 2\delta$. The position limits in Theorem 1 change to $L_1 = \min(L_2 - 1, \frac{2(u_A + \frac{1}{2}M - 1)}{\varepsilon'})$ and $L_2 = \frac{2(R-1) + M - 2\delta}{\varepsilon'}$.

5.3 Finite number of rounds

Assume that the number of trading rounds (N) is a random variable with a geometric distribution $Prob(N = n) = p(1 - p)^{n-1}$ for $n = 1, 2, \dots$, so after each round there is a probability p , independent of history, that trading will stop; assume that an agent does not know the realization of N . In the limit case $p = 0$ (analyzed in previous sections), N approaches infinity, and an agent can enter as many contracts as he wants. However, when $p > 0$, there is a positive probability that an agent who plans a strategic default will not be able to enter as many contracts as he plans, so we need to replace n_i in the incentive constraint with $E[\min(n_i, N)]$. The nature of the results remains (see appendix).²⁷

6 Comparative statics

Collateral and investment. The next proposition provides comparative statics on the optimal amount of collateral k under the third best. The implications for x , I , and $\frac{k}{x}$ follow immediately from the relations $x = \varepsilon - (1 + \varepsilon)k$ and $I = 1 - k$. Proposition 3 provides support as to why it makes sense to focus on third-best contracts when doing comparative statics for the decentralized environment. The implications for the second-best contract are the same as in the third best, but the second-best contract does not depend on δ and p .

Proposition 5 *The optimal amount of collateral (third best) decreases in R (the project's expected return), M (the penalty upon default), δ (the fixed cost per trade), and p (the prob-*

²⁶The second requirement reduces to $\delta < \frac{1}{2}(x_{sb} - k_{sb})$. This ensures that $\bar{U}_i(\psi, n_i)$ in equation (21) is increasing in n_i . When $R \geq 1 + \frac{1}{2}(\varepsilon - M)$, it follows from Proposition 1 that $\frac{1}{2}(x_{sb} - k_{sb}) = \frac{1}{2}\varepsilon$. When $R < 1 + \frac{1}{2}(\varepsilon - M)$, it follows that $\frac{1}{2}(x_{sb} - k_{sb}) = \frac{1}{2}[\varepsilon - (2 + \varepsilon)\frac{\varepsilon - 2(R-1) - M}{\varepsilon - 2R + 4}] = \frac{2(R-1) - M}{\varepsilon - 2R + 4}$.

²⁷With probability p , an agent ends up with only one contract, exactly as suggested by the central planner. Since the agent does not default in this case, it might be optimal to design a contract that does not satisfy the incentive constraint. In other words, it might be optimal to require less collateral knowing that agents will sometimes default, but not always. I rule out this case by assuming that p is small enough.

ability of not finding an additional counterparty), but increases in ε (the project's liquidity need).

The optimal contract requires less collateral when strategic default is less beneficial. This happens when either of the following conditions holds: (1) The penalty for default and/or the expected return on the project are high. (2) Markets are less liquid, in the sense that the fixed cost per trade and/or the probability of not finding a trading counterparty are high. (3) The liquidity need is low.

In my model R affects collateral in two ways: It increases the cost for default (just like M), but it also increases the opportunity cost of posting collateral. According to the model, agents use less collateral and invest more in good times (when R is high), but they use more collateral and invest less in bad times (when R is low). This suggests a propagation mechanism, which is beyond the scope of this paper. The model also predicts that collateral is higher when the moral hazard intensity is higher. Along the lines of Acharya and Viswanathan (2009), the moral hazard intensity is defined here as the spread between the return from investing in one's project and the return from diverting funds to consumption.

Liquid markets present a problem in this model because they create more opportunities for strategic default, thereby forcing agents to post more collateral and invest less. Higher ε increases the gain from strategic default because the promised payment increases.

Position limits. Position limits must be higher when the gain from strategic default is lower. More precisely, the lower bound and upper bound that define the intervals in Theorem 1 and Proposition 4 are higher.

Proposition 6 *The position limits L_1 and L_2 increase in R (the project's expected return), M (the penalty upon default), δ (the fixed cost per trade), and p (the probability of not finding an additional counterparty), but decrease in ε (the project's liquidity need).*

Gains from the optimal mechanism. The gain from the optimal mechanism is $U_i(\psi_{sb}) - U_i(\psi_{tb}) = (R - 1)(k_{tb} - k_{sb})$. Since k_{sb} does not depend on δ and p , but k_{tb} does, it follows from Proposition 5 that the gain increases when δ and/or p fall. Intuitively, when markets are more liquid (in the sense of a lower fixed cost per trade and a lower probability of not finding a counterparty), nonexclusivity is a bigger problem and a mechanism can add more.

Proposition 7 *The gain from the optimal mechanism increases when the fixed cost per trade (δ) and/or the probability of not finding a counterparty (p) decrease.*

7 Conclusion

The paper constructs a mechanism that induces agents to voluntarily reveal to it all the contracts they enter. The mechanism allows each agent to report every contract he enters, and it makes public the names of agents who have reached some prespecified position limit. The main result is that the mechanism achieves the same outcome that could be implemented if agents could not enter contracts secretly. The mechanism does it in a very cost-effective way: It does not need to monitor every possible transaction that an agent can make. It only needs to monitor the contracts that agents choose to report to it. The paper also solves for the best outcome without a mechanism and provide some comparative statics for the optimal amount of collateral that is needed to sustain an equilibrium. It also shows that the gain from the mechanism increases when markets become more liquid.

The paper uses a simple framework, but the main intuition applies in richer settings. For example, if agents could enter more than one type of contract, the mechanism would need to say what an agent's allowable positions are, rather than specify a single position limit. To induce reporting, the allowable position may need to be broad enough, so that an agent has enough scope to default if his counterparty does not report the contract. Then reporting can be induced.

8 Appendix

8.1 Proofs

Lemma 3 Equation (10) can be replaced with

$$U_i(I_i|\psi) \geq U_i(0|\psi). \quad (22)$$

Proof of Lemma 3:

First, equation (2) must be binding, so $I_i = 1 - k_i$; otherwise, we can increase the value of the objective function without violating the constraints, by increasing I_i and k_i by Δ and $\varepsilon\Delta$, respectively, where Δ is small enough. Note that equation (3) then implies that $\frac{x_i - k_i}{\varepsilon} \leq 1 - k_i$.

Next I show that $U(I'_i|\psi)$ obtains a maximum on $[0, 1 - k_i]$ when either $I'_i = 1 - k_i$ (as suggested by the planner) or $I'_i = 0$. Consider $I'_i \in [0, 1 - k_i]$. Equation (4) implies that $\beta_i(I'_i) = 1$, and equation (7) implies that $d(I'_i) = 1$ if and only if $I'_i \geq \frac{x_i - k_i}{\varepsilon}$. Therefore, for every $I'_i \in [0, \frac{x_i - k_i}{\varepsilon})$, equation (8) reduces to

$$\begin{aligned} U(I'_i|\psi) &= (1 - k_i - I'_i) + \frac{1}{2}(\varepsilon I'_i) + \frac{1}{2}[k_i + x_{-i} + (R - \varepsilon)I'_i] \\ &\quad 1 + \frac{1}{2}(x_{-i} - k_i) + (\frac{1}{2}R - 1)I'_i, \end{aligned} \quad (23)$$

and for every $I'_i \in [\frac{x_i - k_i}{\varepsilon}, 1 - k_i]$, the equation reduces to

$$U(I'_i|\psi) = 1 + \frac{1}{2}(x_{-i} - x_i) + (R - 1)I'_i. \quad (24)$$

The function $U(I'_i|\psi)$ is piecewise linear in I'_i with a discontinuity at $I'_i = \frac{x_i - k_i}{\varepsilon}$. The size of the jump is $\frac{1}{2}R\frac{x_i - k_i}{\varepsilon} - \frac{1}{2}(x_i - k_i)$. Since $R > \varepsilon$, the jump is positive. Since $R > 1$, $U(I'_i|\psi)$ is increasing on $[\frac{x_i - k_i}{\varepsilon}, 1 - k_i]$ and does not obtain a maximum on $I'_i = \frac{x_i - k_i}{\varepsilon}$. It must be the case then that $U(I'_i|\psi)$ obtains a maximum when either $I'_i = 0$ or $I'_i = 1 - k_i$. Q.E.D.

Proof of Proposition 1:

I solve the more general case with the added parameters from Section 5. Equation (12) becomes

$$\frac{1}{2}(x_i - k_i) - \frac{1}{2}M \leq (R - 1)I_i. \quad (25)$$

When $R + \frac{1}{2}M \geq 1 + \frac{1}{2}\varepsilon$, the second best equals the first best. Otherwise, the solution is obtained as follows: Equation (2) implies that $I_i = 1 - k_i$. Substituting $I_i = 1 - k_i$ in equations (4) and (25), and rearranging terms, we obtain

$$x_{-i} = \varepsilon - (1 + \varepsilon)k_i \text{ for every } i \in \{1, 2\}, \quad (26)$$

$$x_i = M + 2(R - 1) + (3 - 2R)k_i \text{ for every } i \in \{1, 2\}. \quad (27)$$

From (26) we obtain

$$x_2 - x_1 = (1 + \varepsilon)(k_2 - k_1), \quad (28)$$

and from (27) we obtain

$$x_2 - x_1 = (3 - 2R)(k_2 - k_1). \quad (29)$$

Since $2(1 - R) < 0 < \varepsilon$, it follows that $\varepsilon \neq 2(1 - R)$, and $1 + \varepsilon \neq 3 - 2R$. Thus, equations (28) and (29) imply that $k_1 = k_2$. Denoting $k_i = k$, we obtain from (26) that $x_1 = x_2 = \varepsilon - (1 + \varepsilon)k$, and from (26) and (27) that $\varepsilon - (1 + \varepsilon)k = M + 2(R - 1) + (3 - 2R)k$. Solving for k , we obtain, $k = \frac{\varepsilon - 2R + 2 - M}{\varepsilon - 2R + 4}$. Q.E.D.

Proof of Proposition 2:

I prove the proposition for the more general case in which $U_i(\psi)$ and $\bar{U}_i(\psi, n_i)$ are given in equations (20) and (21). The incentive constraint is then

$$\frac{1}{2}(x_i - x_{-i}) + \frac{1}{2}n_i(x_{-i} - k_i) - (n_i - 1)\delta \leq (R - 1)I_i + \frac{1}{2}M. \quad (30)$$

As in Lemma 3, equation (2) must be binding. Substituting $I_i = 1 - k_i$ and $n_i = \frac{1}{k_i}$ in equation (30), summing over $i = 1, 2$, and rearranging terms, we obtain that

$$(R - 1) \sum_{i=1}^2 (1 - k_i) + M + \delta \sum_{i=1}^2 \left(\frac{1}{k_i} - 1 \right) \geq \frac{1}{2} \sum_{i=1}^2 \left(\frac{x_{-i}}{k_i} - 1 \right). \quad (31)$$

From equation (4) we obtain that

$$\sum_{i=1}^2 (k_i + x_i) \geq \sum_{i=1}^2 \varepsilon (1 - k_i). \quad (32)$$

To prove the proposition, it is enough to show that the contract in the proposition is feasible, in Ψ , and is the a unique solution to $\min(k_1 + k_2)$ subject to equations (31) and (32). To show the last part, I use the KKT conditions, as explained below. The rest follows easily.

Let $f(\psi) = k_1 + k_2$, and consider the problem $\min f(\psi)$ subject to $g_i(\psi) \leq 0$ for every $i \in \{1, 2\}$, where

$$g_1(\psi) \equiv \frac{1}{2} \sum_{i=1}^2 \left(\frac{x_{-i}}{k_i} - 1 \right) - (R-1) \sum_{i=1}^2 (1 - k_i) - M - \delta \sum_{i=1}^2 \left(\frac{1}{k_i} - 1 \right), \quad (33)$$

$$g_2(\psi) \equiv \varepsilon \sum_{i=1}^2 (1 - k_i) - \sum_{i=1}^2 (k_i + x_i). \quad (34)$$

Denote the Lagrange multiplier of $g_i(\psi)$ by λ_i , and let $L(\psi) = f(\psi) + \sum_{j=1}^2 \lambda_j g_j(\psi)$. An (optimal) solution must satisfy $\lambda_i \geq 0$,

$$\frac{\delta L}{\delta x_i} = \frac{\lambda_1}{2k_{-i}} - \lambda_2 = 0 \quad (35)$$

and

$$\frac{\delta L}{\delta k_i} = 1 - \frac{x_{-i}}{2k_i^2} \lambda_1 + (R-1)\lambda_1 + \delta \frac{1}{k_i^2} \lambda_1 - \varepsilon \lambda_2 - \lambda_2 = 0. \quad (36)$$

Equation (35) implies that $k_1 = k_2 \equiv k$. Equation (36) then implies that $x_1 = x_2 \equiv x$. If $\lambda_2 = 0$, equation (35) implies $\lambda_1 = 0$, but this contradicts equation (36). Therefore, $\lambda_2 > 0$. Equation (35) then implies that $\lambda_1 > 0$, so equations (31) and (32) are binding. From equation (32), we obtain that $x = \varepsilon - (1 + \varepsilon)k$. Equation (31) then reduces to

$$\frac{\varepsilon - (1 + \varepsilon)k}{k} - 1 = 2r(1 - k) + M + 2\delta \left(\frac{1}{k} - 1 \right), \quad (37)$$

where $r = R - 1$. This further reduces to

$$2rk^2 - bk + \varepsilon' = 0, \quad (38)$$

where $b = 2 + \varepsilon' + 2r + M$, and $\varepsilon' = \varepsilon - 2\delta$. (Footnote 26 implies that $\varepsilon' > 0$.) Equation (38) has two roots:

$$k_1 = \frac{1}{4r} \left(b + \sqrt{b^2 - 8r\varepsilon'} \right) \geq 1 \quad (39)$$

$$k_2 = \frac{1}{4r} \left(b - \sqrt{b^2 - 8r\varepsilon'} \right) < 1. \quad (40)$$

One can show that the two solutions satisfy linear independence constraint qualification, and that a contract that does not satisfy this regularity assumption is suboptimal. To see that $k_1 \geq 1$ (so k_1 is suboptimal), note that

$$\begin{aligned} k_1 &> \frac{1}{4r} \left(\varepsilon' + 2r + \sqrt{(\varepsilon' + 2r)^2 - 8r\varepsilon'} \right) \\ &= \frac{1}{4r} (\varepsilon' + 2r + \sqrt{(\varepsilon' - 2r)^2}). \end{aligned} \quad (41)$$

If $\varepsilon' \geq 2r$, we obtain that $k_1 > \frac{1}{4r}(\varepsilon' + 2r + \varepsilon' - 2r) = \frac{\varepsilon'}{2r} \geq 1$. If $\varepsilon' < 2r$ we obtain that $k_1 > \frac{1}{4r}(\varepsilon' + 2r - (\varepsilon' - 2r)) = 1$. To see that $k_2 < 1$ (so it is the unique solution), note that $k_2 < 1$ is equivalent to $b - 4r < \sqrt{b^2 - 8r\varepsilon'}$. If $b < 4r$, the result follows immediately because the left-hand side is negative and the right-hand side is positive. Otherwise, we need to show that $(b - 4r)^2 < b^2 - 8r\varepsilon'$, which is equivalent to $b > 2r + \varepsilon'$. The last inequality follows from the definition of b . Finally, to show that $k_2 > 0$, note that $\sqrt{b^2 - 8r\varepsilon'} < \sqrt{b^2} = b$. Q.E.D.

Proof of Lemma 1:

Suppose that $\rho_1 = 1$ and $\rho_2 = 0$. (The case $\rho_1 = 0$, $\rho_2 = 1$ is symmetric.) Choosing $d_1 = 0$ is a credible threat if either $(d_1 = d_2 = 0)$ or $(d_1 = 0, d_2 = 1)$ is an equilibrium. Since observing $\rho_2 = 0$ is off the equilibrium path, we can assign any beliefs to agent 1. The calculations below assume that agent 1 believes that all the other agents who are present in this round have just showed up for trade. Other beliefs will reduce his gain from strategic default and will make it harder to sustain an equilibrium.

Consider first $d_1 = d_2 = 0$. Choosing $d_1 = 0$ is a best response against $d_2 = 0$ if $\bar{U}_i(\psi_{sb}, n(L, 0)) - \frac{1}{2}(x_{sb} - k_{sb}) \geq U_A - \frac{1}{2}(x_{sb} - k_{sb})$. This reduces to $\bar{U}_i(\psi_{sb}, L + 1) \geq U_A$. Choosing $d_2 = 0$ is a best response for agent 2 if $\bar{U}_i(\psi_{sb}, n(L, 1)) - \frac{1}{2}(x_{sb} - k_{sb}) \geq U_A - \frac{1}{2}(x_{sb} - k_{sb})$. This reduces to $\bar{U}_i(\psi_{sb}, L) \geq U_A$. Since $\bar{U}_i(\psi_{sb}, L + 1) > \bar{U}_i(\psi_{sb}, L)$, the above is an equilibrium if

$$\bar{U}_i(\psi_{sb}, L) \geq U_A. \quad (42)$$

Consider now $d_1 = 0, d_2 = 1$. Choosing $d_1 = 0$ is a best response against $d_2 = 1$ if

$$\bar{U}_i(\psi_{sb}, n(L, 0)) \geq U_i(\psi_{sb}). \quad (43)$$

Choosing $d_2 = 1$ is a best response against $d_1 = 0$ if $U_A - \frac{1}{2}(x_{sb} - k_{sb}) \geq \bar{U}_i(\psi_{sb}, n(L, 1)) - \frac{1}{2}(x_{sb} - k_{sb})$, which reduces to

$$U_A \geq \bar{U}_i(\psi_{sb}, L). \quad (44)$$

Thus, the above is an equilibrium if equations (43) and (44) hold.

If $R < 1 + \frac{1}{2}\varepsilon$, then for every $L \geq 1$, $\bar{U}_i(\psi_{sb}, L) \geq \bar{U}_i(\psi_{sb}, 1) = U_i(\psi_{sb}) > U_A$. Thus, equation (42) holds for every $L \geq 1$, and the threat of default is credible.

Otherwise ($R \geq 1 + \frac{1}{2}\varepsilon$), equation (42) is equivalent to $1 + \frac{1}{2}\varepsilon L \geq U_A$, which reduces to $L \geq \frac{2(U_A-1)}{\varepsilon} = \frac{2(R-1)}{\varepsilon(1+\varepsilon)} \equiv L_3$. Equation (44) is equivalent to $L \leq L_3$, and equation (43) is equivalent to $1 + \frac{1}{2}(L+1)\varepsilon \geq R$, which reduces to $L \geq L_2 - 1$.

If $L_3 < L_2 - 1$, the first equilibrium exists if $L \geq L_3$, whereas the second equilibrium does not exist for any L . If $L_3 \geq L_2 - 1$, the first equilibrium exists if $L \geq L_3$ and the second equilibrium exists if $L \in [L_2 - 1, L_3]$. Thus, when $L \geq \min(L_3, L_2 - 1)$ at least one of the two equilibria above exist and the threat of default is credible. Q.E.D.

Proof of Corollary 1:

The position limit is nonbinding in equilibrium if $L_2 - 1 > 1$ and $\frac{2(U_A-1)}{\varepsilon} = \frac{2(R-1)}{\varepsilon(1+\varepsilon)} > 1$. Since $L_2 = \frac{2(R-1)}{\varepsilon}$, it follows that $L_2 - 1 > 1$ if and only if $\frac{2(R-1)}{\varepsilon} > 2$. This is equivalent to $R > 1 + \varepsilon$. The second condition ($\frac{2(R-1)}{\varepsilon(1+\varepsilon)} > 1$) holds if and only if $R > 1 + \frac{1}{2}\varepsilon + \frac{1}{2}\varepsilon^2$. Since $\varepsilon < 1$, it follows that $1 + \frac{1}{2}\varepsilon + \frac{1}{2}\varepsilon^2 < 1 + \frac{1}{2}\varepsilon + \frac{1}{2}\varepsilon = 1 + \varepsilon$. Thus, the first condition implies the second condition. Q.E.D.

Proof of Proposition 3

Denote

$$\begin{aligned} \bar{U}_i(\psi', \psi, n_i) &= (1 - k'_i - n_i k_i) + \frac{1}{2}(k'_i + x'_{-i}) + \frac{1}{2}n_i(k_i + x_{-i}) \\ &= 1 + \frac{1}{2}(x'_{-i} - k'_i) + \frac{1}{2}n_i(x_{-i} - k_i). \end{aligned} \quad (45)$$

Analogue to equation (14), this is the utility for an agent of type i who plans a strategic default by entering ψ' with one counterparty and ψ with n counterparties. Such a deviation is feasible only if $k'_i + n_i k_i \leq 1$. I use the following lemma to prove the proposition:

Lemma 4 *A feasible contract $\psi \in \Psi$ is renegotiation proof if and only if there does not exist a feasible contract ψ' , such that for $i \in \{1, 2\}$:*

$$U_i(\psi') \geq U_i(\psi), \text{ with strict inequality for at least one } i \quad (46)$$

$$U_i(\psi') \geq \bar{U}_i(\psi', \psi, n_i) \text{ for every } n_i \in [0, \frac{1 - k'_i}{k_i}]. \quad (47)$$

Proof of Lemma 4. Denote by $F(\psi)$ the set of feasible contracts ψ' that satisfy equations (46) and (47) for $i \in \{1, 2\}$. We need to show that a feasible $\psi \in \Psi$ is renegotiation proof if and only if $F(\psi)$ is empty.

Consider a feasible contract $\psi \in \Psi$.

Only if: Suppose by contradiction that ψ is renegotiation proof and there exists $\psi' \in F(\psi)$. Equation (46) implies that $\psi' \neq \psi$. I show that the game G_2 has a PBE whose outcome is that all agents enter ψ and the randomly chosen pair enters ψ' , which contradicts the fact that ψ is renegotiation proof. Equilibrium strategies and beliefs are as in footnote 17 with the following addition: An agent does not change his beliefs after being offered a contract by the side planner. If the side planner suggests ψ' , an agent accepts it. If the side planner offers any other contract, the agent does not accept. After entering ψ' , the agent does not enter additional contracts. Since $\psi' \in F(\psi)$, the strategies and beliefs above are a PBE, and since the probability that agents are chosen by the side planner is zero, the possibility of renegotiation does not affect agents' decisions prior to being chosen.

If: Suppose by contradiction that $F(\psi)$ is empty and $\psi \in \Psi$ is not renegotiation proof. Then the game G_2 has a PBE whose outcome is that all agents enter ψ and the randomly chosen pair enters $\psi' \neq \psi$. To obtain this equilibrium path it must be that the side planner offers ψ' and the two agents accept. Consistent with this equilibrium path, an agent who is offered ψ' by the side planner does not revise his beliefs (i.e., he believes all other agents

including his counterparty have just shown up for trade), and accepting ψ' is optimal only if $\psi' \in F(\psi)$. But this contradicts the fact that $F(\psi)$ is empty. Q.E.D.

To continue the proof of the proposition, note that when $x_{-i} > k_i$, equation (47) reduces to

$$(R-1)I'_i \geq \frac{1}{2}(x'_i - k'_i) + \frac{1}{2}(1 - k'_i)\left(\frac{x_{-i} - k_i}{k_i}\right). \quad (48)$$

Part 1. According to Lemma 4, it is enough to show that ψ_{tb} is the unique solution to the following problem: Find a feasible contract that maximizes $I_1 + I_2$ subject to

$$(R-1)I_i \geq \frac{1}{2}(x_i - k_i) + \frac{1}{2}(1 - k_i)\left(\frac{x_{tb} - k_{tb}}{k_{tb}}\right). \quad (49)$$

for $i \in \{1, 2\}$.

This is a linear programming problem. Since ψ_{tb} satisfies equation (16) with equality, it also satisfies equation (49) with equality and is the unique solution.

Part 2. Suppose by contradiction that there is a feasible and symmetric contract $\psi = (k, x, I) \in \Psi$, such that ψ is renegotiation proof and $\psi \neq \psi_{tb}$. Without loss of generality, $k + I = 1$; otherwise, we can increase I and k by Δ and $\varepsilon\Delta$, respectively, and create a feasible contract ψ' that satisfies the conditions in Lemma 4, in contradiction to the fact that ψ is renegotiation proof.

Since ψ satisfies equation (16) and is symmetric, it follows that

$$\frac{1}{2} \frac{1}{k}(x - k) \leq (R-1)I, \quad (50)$$

and since ψ_{tb} satisfies equation (16) with equality, it follows that

$$\frac{1}{2} \frac{1}{k_{tb}}(x_{tb} - k_{tb}) = (R-1)I_{tb}. \quad (51)$$

Since ψ_{tb} is a unique solution to the third-best problem, we must have $I < I_{tb}$ (and also

$k_{tb} < k$), and it follows from (50) and (51) that $(x_{tb} - k_{tb})/k_{tb} > (x - k)/k$, and

$$\begin{aligned}
(R-1)I_{tb} &= \frac{1}{2} \frac{1}{k_{tb}} (x_{tb} - k_{tb}) \\
&= \frac{1}{2} (x_{tb} - k_{tb}) + \frac{1}{2} (1 - k_{tb}) \frac{x_{tb} - k_{tb}}{k_{tb}} \\
&> \frac{1}{2} (x_{tb} - k_{tb}) + \frac{1}{2} (1 - k_{tb}) \frac{x - k}{k} \\
&> \frac{1}{2} (x_{tb} - k_{tb}) + \frac{1}{2} (1 - k) \frac{x - k}{k}
\end{aligned} \tag{52}$$

But then ψ_{tb} satisfies the conditions in Lemma 4 in contradiction to the fact that ψ is renegotiation proof. Q.E.D.

Proof of Lemma 2:

Suppose that $\rho_1 = \rho_2 = 0$. Choosing $d_1 = 1$ is a best response against $d_2 = 1$, and vice versa, if $U_i(\psi_{sb}) \geq \bar{U}_i(\psi_{sb}, n(L, 0))$. When $R < 1 + \frac{1}{2}\varepsilon$, this condition is violated for every $L > 0$. Otherwise ($R \geq 1 + \frac{1}{2}\varepsilon$), the condition simplifies to $R \geq 1 + \frac{1}{2}(L+1)\varepsilon$, which reduces to $L \leq L_2 - 1$. Q.E.D.

Finite number of rounds

When N is finite, an agent who plans to enter n contracts can enter only $\min(n, N)$ contracts. Since N has a geometric distribution, it follows that $E \min(n, N) = \frac{1}{p}[1 - (1-p)^n]$. This is because $\min(n, N) = N - \max(N - n, 0)$, $E(N) = \frac{1}{p}$ and

$$\begin{aligned}
E \max(N - n, 0) &= \sum_{i=n+1}^{\infty} (i - n)p(1-p)^{i-1} \\
&= (1-p)^n \sum_{i=n+1}^{\infty} (i - n)p(1-p)^{i-n-1} = (1-p)^n \frac{1}{p}.
\end{aligned} \tag{53}$$

The expression for $\bar{U}_i(\psi, n_i)$ becomes

$$\bar{U}_i(\psi, n_i) = 1 + \frac{1}{2p}[1 - (1-p)^{n_i}](x_{-i} - k_i - 2\delta) - \frac{1}{2}M. \tag{54}$$

Denote $f(p, k) \equiv \frac{1}{p}[1 - (1-p)^{1/k}]$. The incentive constraint is

$$\frac{1}{2}(x_i - x_{-i}) + \frac{1}{2}f(p, k_i)(x_{-i} - k_i - 2\delta) + \delta \leq (R-1)I_i + \frac{1}{2}M. \tag{55}$$

Using steps similar to Proposition 2, we can show that $k_1 = k_2 = k$, $x_1 = x_2 = \varepsilon - (1 + \varepsilon)k$, and equations (2), (55), and (32) must be binding. [In more details, instead of equation (31) we obtain

$$(R - 1) \sum_{i=1}^2 (1 - k_i) + M \geq 2\delta + \frac{1}{2} \sum_{i=1}^2 f(p, k_i)(x_{-i} - k_i - 2\delta), \quad (56)$$

and $g_1(\psi)$ changes to

$$g_1(\psi) \equiv \frac{1}{2} \sum_{i=1}^2 f(p, k_i)(x_{-i} - k_i - 2\delta) - (R - 1) \sum_{i=1}^2 (1 - k_i) - M + 2\delta. \quad (57)$$

$$\frac{\delta L}{\delta x_i} = \lambda_1 \frac{1}{2} f(p, k_{-i}) - \lambda_2 = 0 \quad (58)$$

$$\frac{\delta L}{\delta k_i} = 1 - \frac{1}{2} \lambda_1 f(p, k_i) + \frac{1}{2} \lambda_1 (x_{-i} - k_i - 2\delta) \frac{\partial}{\partial k} f(p, k_i) - \varepsilon \lambda_2 - \lambda_2 = 0. \quad (59)$$

Equation (55) reduces to

$$f(p, k)[\varepsilon - (2 + \varepsilon)k - 2\delta] + 2\delta - M = 2r(1 - k) \quad (60)$$

When p is small enough, the equation has a unique solution $k \in (0, 1)$. To see why, denote the right-hand side by $A(k)$ and the left-hand side by $B(k)$, and note that both sides are continuous and strictly decreasing in k , $\lim_{k \downarrow 0} A(k) > B(0)$, and $A(1) < B(1)$. Q.E.D.

Proof of Proposition 5:

From the proof of Proposition 2 we know that the optimal amount of collateral solves $H = 0$, where $H = 2rk^2 - bk + \varepsilon'$. Comparative statics can be obtained by taking partial derivatives (e.g., $\frac{\partial k}{\partial \varepsilon} = -\frac{\partial H / \partial \varepsilon}{\partial H / \partial k}$), and using the fact that $k = \frac{1}{4r} \left(b - \sqrt{b^2 - 8r\varepsilon'} \right)$ to show that $\frac{\partial H}{\partial k} = 4rk - b = -\sqrt{b^2 - 8r\varepsilon'} < 0$. When the number of rounds is finite, use $H = A - B$ (as defined above). To show that $\frac{\partial H}{\partial k} < 0$, note that since $A''(k) > 0$, it follows that $A'(k) < B'(k)$. [To see that $A''(k) > 0$, note that $f'(k) < 0$, $f''(k) > 0$, $A'(k) = f'(k)[\varepsilon - (2 + \varepsilon)k - 2\delta] - f(k)(2 + \varepsilon)$, and $A''(k) = f''(k)[\varepsilon - (2 + \varepsilon)k - 2\delta] - 2f'(k)(2 + \varepsilon)$.] Q.E.D.

Proof of Proposition 6:

When $R \geq 1 + \frac{1}{2}(\varepsilon - M)$, the results follow from taking partial derivatives to $L_3 = \frac{2(u_A + \frac{1}{2}M - 1)}{\varepsilon'}$ and $L_2 = \frac{2(R-1) + M - 2\delta}{\varepsilon'}$ ($L_1 = \min(L_2 - 1, L_3)$). (Note that $\frac{\partial L_2}{\partial \delta} = \frac{-2(\varepsilon - 2\delta) + 2[2(R-1) + M - 2\delta]}{(\varepsilon - 2\delta)^2} = \frac{-2\varepsilon + 4(R-1) + 2M}{(\varepsilon - 2\delta)^2} \geq \frac{2M}{(\varepsilon - 2\delta)^2}$, where the last inequality follows since $R \geq 1 + \frac{1}{2}\varepsilon$.)

When $p > 0$, the expressions for L_3 and L_2 are obtained from the equations $\frac{1}{p}[1 - (1 - p)L_3] = \frac{2(u_A + \frac{1}{2}M - 1)}{\varepsilon'}$ and $\frac{1}{p}[1 - (1 - p)L_2] = \frac{2(R - 1) + M - 2\delta}{\varepsilon'}$. Comparative statics are obtained from taking partial derivatives. For example, let $H = \frac{1}{p}[1 - (1 - p)L_3] - \frac{2(u_A + \frac{1}{2}M - 1)}{\varepsilon'}$. Then $\frac{\partial L_3}{\partial p} = -\frac{\partial H / \partial p}{\partial H / \partial L_3} > 0$ since $\frac{\partial H}{\partial p} < 0$ and $\frac{\partial H}{\partial L_3} > 0$. Q.E.D.

8.2 A formal definition of renegotiation proofness

To define renegotiation proofness formally, I modify the message game (G_0) as follows: The sequence of events in the modified game is as in G_0 , but I add the following step 2.5 between step 2 and step 3:

Step 2.5: A pair of agents is chosen randomly and a side planner suggests that they enter ψ' . If both agents accept (they decide simultaneously), they enter ψ' instead of ψ (the contract set by the planner). Otherwise, they stick with ψ . (ψ' can be different from ψ but it can also equal ψ .)

Denote the modified game above by G_2 , and denote the same game that does not include steps 1, 3, and 4 in each round by G_3 . The game G_2 corresponds to the message game with the mechanism, and G_3 corresponds to the decentralized setting without the mechanism. Note that it does not matter whether we randomly choose one pair in each round or one pair in the whole game because the probability of being chosen twice or meeting another agent who was chosen is zero.

Definition 1 *A contract $\psi \in \Psi$ is renegotiation proof in the decentralized environment if the modified game G_3 does not have a PBE whose outcome is that all agents enter ψ , and the randomly chosen pair enters $\psi' \neq \psi$.*

Definition 2 *Reporting a contract $\psi \in \Psi$ is renegotiation proof if the modified game G_2 does not have a PBE whose outcome is that either (i) all agents enter ψ and report it and the randomly chosen pair enters ψ without reporting; or (ii) all agents enter ψ and report it and the randomly chosen pair enters $\psi' \neq \psi$ with or without reporting.*

I say that an outcome ψ can be implemented in the decentralized economy in a renegotiation proof way if ψ can be implemented and ψ is renegotiation proof. Similarly, a mechanism can implement ψ in a renegotiation proof way if it can implement ψ and reporting ψ is renegotiation proof.

Note that renegotiation here does not occur because of an arrival of new information. It is just a way to capture the idea that each pair of agents can coordinate their actions in a self-enforcing way to do the best for them given that all other agents stick with their equilibrium strategies. In addition, a side planner is not crucial. We can assume that the two agents renegotiate directly; for example, they both offer new contracts simultaneously, or one agent offers to change the contract and the other agent accepts or rejects.²⁸

²⁸The definitions above are in the spirit of Laffont and Martimort (1997). They use the term *collusion proof* to model collusion between two firms with private information about their costs. In their setting a regulator offers a mechanism (a grand contract), and an uninformed third party then offers a side contract.

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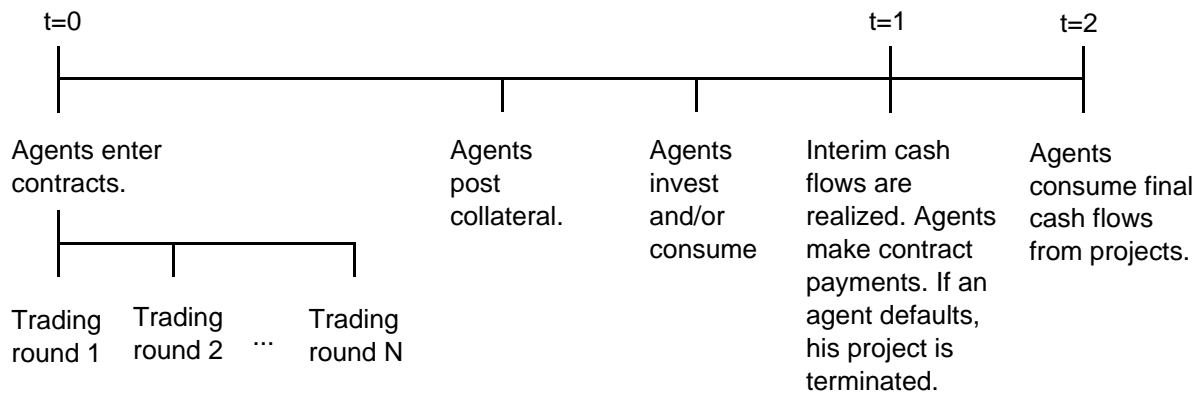


Figure 2: Time Line

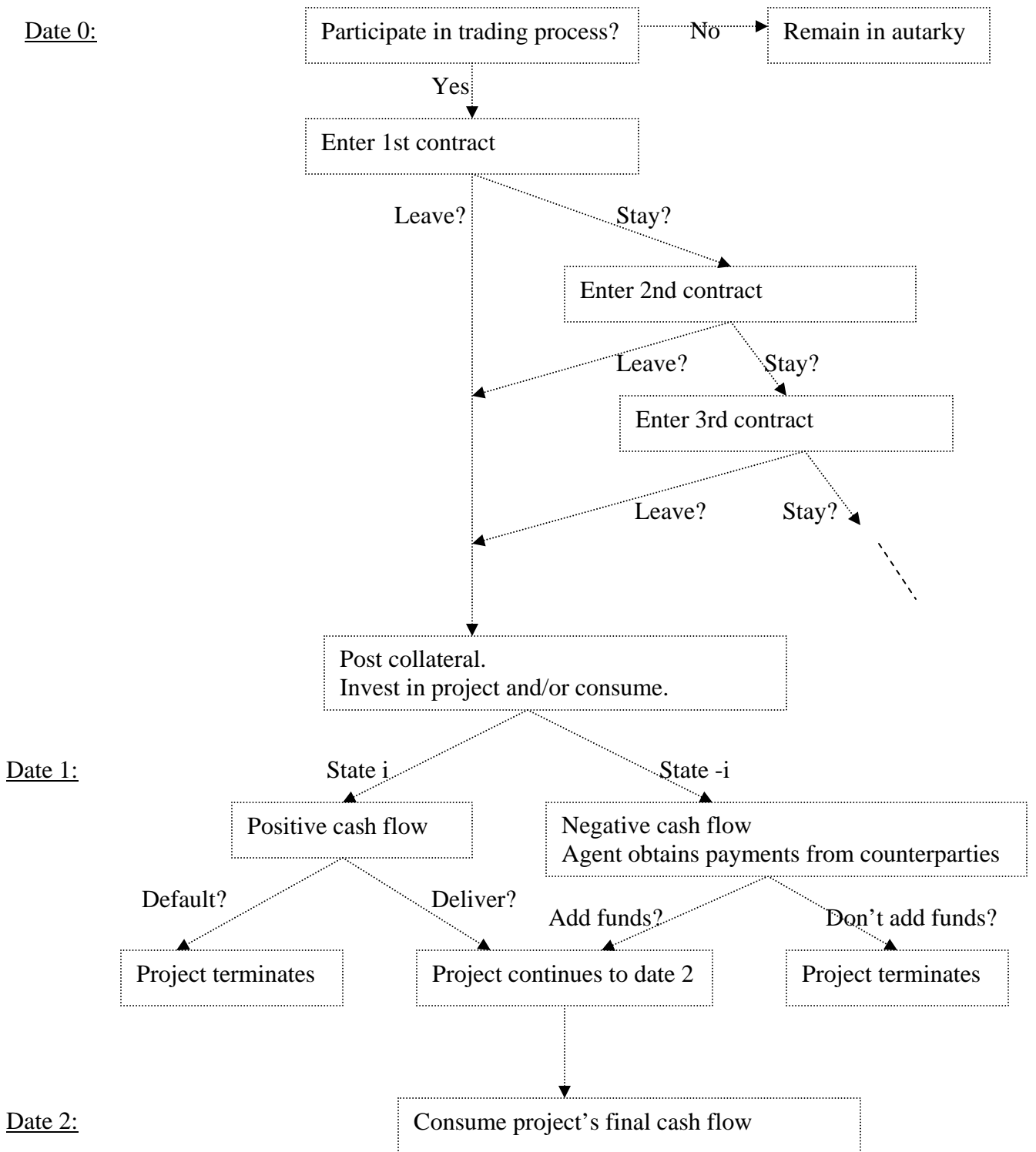


Figure 3: Sequence of events for an agent of type i.